MINIMUM DISTANCE ESTIMATION
ON TIME SERIES ANALYSIS
WITH LITTLE DATA

THESIS

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Table of Contents

Page

List of Figures

Page

List of Tables

Page

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Abstract

Minimum distance estimate is a statistical parameter estimate technique that selects model parameters that minimize a good-of-fit statistic. Minimum distance estimation has been demonstrated better standard approaches, including maximum likelihood estimators and least squares, in estimating statistical distribution parameters with very small data sets. This research applies minimum distance estimation to the task of making time series predictions with very few historical observations. In a Monte Carlo analysis, we test a variety of distance measures and report the results based on many different criteria. Our analysis tests the robustness of the approach by testing its ability to make predictions when the fitted time-series model does not match the data generation model. Our analysis indicates benefits in applying minimum distance estimation when making time series prediction based on less than 30 observations.

MINIMUM DISTANCE ESTIMATION

FOR TIME SERIES ANALYSIS WITH LITTLE DATA

I. INTRODUCTION

Forecasting is very important in many types of organizations, since prediction of the future must be incorporated into the decision making process [6:3]. In generating forecasts of events that will occur in the future, a forecaster must rely on information concerning events that have occurred in the past. Quantitative forecasting methods are applied when a sequence of numerical observations are recorded sequentially in time. Time series analysis is forecasting techniques that base forecasts only on past observations in that sequence.

Time series examples occur in a variety of fields, ranging from economics to engineering. A monthly sequence of the quantity of parts ordered; a weekly series of the number of road accidents; hourly observations made on the yield of a chemical process are examples of data that appear as *time series*. Given an observed time series, one may wish to predict future values of the series. For instance, the government of a country must be able to predict inflation rates based on past inflation rates over time; a personnel manager must be able to forecast the number of workers required in different job categories based on recent recruiting data in order to plan the supplying of labor and training programs; or Air Force Logistics Command must predict the demand for each part in its inventory for specific time periods to plan purchasing schedules from contractors and inventory maintenance.

To make all of these predictions, the forecasters analyzes past data in order to identify a *pattern* or *model* that can be used to describe the data. After identifying the model, it then becomes possible to estimate parameters. Then this model is extrapolated or extended into the

future to prepare a forecast, which clearly rests on the assumption that the pattern that has been identified will continue in the future.

After having the data sample set from the specified model, statistical theory approaches the estimation by trying to determine the properties of the parameters of the specified model. In this research, the specified model is the *autoregressive moving average* (ARMA) for time series analysis. Several estimation techniques can be found to estimate the parameters of an ARMA model. Some of the well-known estimating methods are the least squares, and the maximum likelihood.

Although the dramatic increase of computer storage enables us to store large amounts of data, many forecasters need to make predictions with few past observations. Often decision makers want to predict future values of a newly developed or implemented system. Since the system is new, very few past observations are available. For example, due to the high cost of collecting data or limited time, most of the new weapon systems have limited data to predict future component failure. However, many of the current forecasting techniques require very large sample sizes with a minimum of 50 to 100 observations to produce good estimates. If the problem involves monthly data, a forecaster would need at least 8 years of data to approach 50 observations and make reliable estimates.

Minimum distance estimation is a statistical parameter estimation technique that selects model parameters to minimize a goodness-of-fit statistic. Several researchers have demonstrated that the minimum distance estimates are better than the least squares and maximum likelihood estimators at estimating statistical distribution parameters for very small sample sizes. We apply minimum distance estimation concept to select time series model parameters for ARMA models when very few historical observations are available. We tested new distance measuring functions and report the results based on various different criteria. We test the robustness of the approach

by evaluating its ability to make predictions when the fitted time series model does not match the data generation model.

The goal of the research is to derive a new parameter estimation technique and to provide a useful tool to forecasters when less then 30 observations are available. Air Force analysts may use minimum distance approach when the available monthly data covers less than 2.5 years (30 months).

Chapter II presents the Box-Jenkins or ARMA models along with a survey on minimum distance estimation with small sample size methods. Chapter III develops and justifies several objective functions. The construction of estimation functions and the methodology of the comparisons are described in detail. Chapter IV outlines the Monte Carlo comparison study and presents our findings. The simulation results are reported, analyzed and the preferred estimation method is identified. Chapter V summarizes our research and makes recommendations for further research.

II. LITERATURE REVIEW

2.1. Introduction

One of the fundamental tasks of engineering and science, and indeed of mankind in general, is the extraction of information from data. Parameter estimation is a discipline that provides tools for the efficient use of data in the estimation of constants appearing in mathematical models and for aiding in modeling of phenomena [3:1]. The derivations and application of modern estimators and estimation algorithms are in the technical literature on communication theory, statistics, control theory, and other fields. Estimation theory originated with the broad area of statistics and gradually found its way through many disciplines of science and engineering [23].

This research project focuses on using minimum distance estimation by minimizing goodness-of-fit statistics for the parameter estimation of ARMA models. Thus, it is important to locate evidence to determine whether minimum distance estimation has advantages over other estimation techniques. To that end, it is necessary to present ARMA models and review the literature in the areas of parameter estimation and goodness-of-fit statistic.

2.2. ARMA Models

Publication of Box and Jenkins' [4] text on forecasting and control techniques with autoregressive moving average (ARMA) processes has had an impact on statistical time series analysis [11:23]. ARMA models have great potential value in the planning environment because they efficiently use prior information. One of many successful application areas of ARMA procedures appears in models by Dent and Swanson [12] on trailer and flatcar loadings.

2.2.1. Autoregressive Models

In the autoregressive model, the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock ε_t [5:9]. If we denote the values of a process at equally spaced times *t, t-1, t-2,* ..., we can represent the model in the form

$$X_t = \mu' + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t \tag{2.1}$$

and it is called an *autoregressive*(AR) process of order *p*.

2.2.2. Moving Average Models

If the current value of the process is linearly dependent on a finite *q* number of previous errors ε_t, the model is called a *moving average* (MA) process of order *q*, and can be represented as

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} \tag{2.2}$$

2.2.3. Mixed Autoregressive - Moving Average (ARMA) Models

Box, Jenkins, and Reinsel [5:11] include both autoregressive and moving average terms in the model to achieve greater flexibility in fitting of actual time series. The basic form of ARMA process can be represented as:

$$X_t = \mu' + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q} \tag{2.3}$$

where ε_t is a random shock at time *t*, with mean zero and unknown variance σ^2 and uncorrelated with ε_s, $s \neq t$ that arises from a white noise process and, although not specifically stated, white noises are normally distributed. The unknown parameters $\mu', \phi_1, \ldots, \phi_p$ and $\theta_1, \ldots, \theta_q$ are estimated from the data.

The ARMA model order is defined by studying the general appearance of the estimated autocorrelation and partial autocorrelation functions [5:185].

2.3. Parameter Estimation

Once we have identified a possible ARMA model and its order, we need to fit that model to the data (which is to estimate the AR and MA parameters for that model) and assess the goodness-of-fit. Most software implementations of the Box-Jenkins method utilize numerical methods to search for the values of the model parameters that minimize the sum of squared residuals.

Most numerical search routines require initial estimates of the parameters to be specified. These initial estimates are most often based on the sample autocorrelations which, in turn, are used in conjunction with the Yule-Walker equations and/or the formulas for the lag-k autocorrelations for an MA(q) process to solve for initial estimates of the autoregressive and moving average parameters.

Box, Jenkins, and Reinsel [5:215] establish that there is only one stationary and invertible model for any specific ARMA(p,q) process. The dependence upon the assumed starting values results in two estimation approaches. First is a conditional estimate, which specifies the starting values, perhaps with their expected value. However, Box, Jenkins, and Reinsel [5:227] state that using the expected value may induce long transient and large estimation errors. In addition, they recommend removing the parameter μ by subtracting the series sample mean from each observation. They state further that the conditional maximum likelihood estimates are the same as the conditional least squares estimates. They recommend assuming the prior shocks are zero, or not including the initial data in the sum of squares until sufficient data exists to calculate the shocks as conditional estimation approaches.

The second approach is an unconditional estimate, which does not assume starting values. Unconditional estimates begin by backforecasting prior to the original data series until these

backforecasts decay to the sample mean. The unconditional estimate is the same as the conditional except it treats the backforecasts and the original series as data.

Three types of estimation technique will be discussed in the following chapters.

2.3.1. Least Squares Estimation

Least squares estimation is the easiest and most common because it simply conducts a nonlinear search within the feasible range of parameters to minimize

$$\sum_{t=-b}^{t=n} \varepsilon_t^2 \tag{2.4}$$

where b indicates the number of residual (error) terms resulting from backforecasts and n is the number of original data values.

Box, Jenkins, and Reinsel [5:307] suggest that least squares must be constrained to the invertible range; otherwise, unrealistic parameter estimates may be obtained that actually provide a smaller sum of squared errors.

For long data series, the squared error term is dominant in the maximum likelihood estimate (MLE). Therefore, the least squares estimates while simple are asymptotically equivalent to MLE parameters. For small to moderate sample sizes, MLE has been demonstrated to be superior to the least squares estimate (particularly with a moving average parameters near the stationary boundaries).

2.3.2. Maximum Likelihood Estimators (MLE)

The MLE parameter estimates maximize the likelihood of the realized time series. Box, Jenkins, and Reinsel [5:225] point out that in the classical view, given the selected model in the correct form, all the information that the data has to tell us about the parameters is contained in

the likelihood function. Furthermore, in the Bayesian view, it is the component of the posterior distribution of the parameters that comes from the data.

Under the assumption that the errors (residuals or shocks) follow a Normal distribution, the likelihood function is

$$L(\varphi,\theta,\sigma_{\varepsilon}) = g(\varphi,\theta,\sigma_{\varepsilon})\exp\left[\frac{-1}{2\sigma_{\varepsilon}^{2}}\left(\sum_{t=-h}^{t=n}\varepsilon_{t}^{2}\right)\right] \tag{2.5}$$

where the function $g(\varphi,\theta,\sigma_{\varepsilon})$ depends upon the specific ARMA model. Abraham and Ledolter [1:250-252] show the maximum likelihood estimator functions for the AR(1), MA(1), and ARMA(1,1) models.

A nonlinear search of the parameter space is conducted to find the maximum values of the functions. The corresponding parameters are the maximum likelihood estimates. The state space formulation and the Kalman filter provide recursive algorithms to calculate maximum likelihood estimators. Gardner, Harvey and Phillips [14] present an algorithm that enables the exact likelihood function of a stationary ARMA process by means of the Kalman filter. Box, Jenkins, and Reinsel [5:237] state that generally, the conditional and unconditional least squares estimators serve as satisfactory approximations to the maximum likelihood estimators for large sample sizes. Hillmer and Tiao [16] develop exact maximum likelihood estimation procedures for stationary mixed ARMA models. Dent and Min [11] examine and compare unconditional least squares estimators, approximate maximum likelihood estimators, Yule-Walker estimators, and regression estimators, and advise the use of maximum likelihood estimation.

2.3.3. Minimum Distance Estimation (MDE)

Wolfowitz [29 and 30] proposed minimum distance estimation as a method to estimate parameters of a statistical distribution. As the name implies, the technique selects the parameters that minimize the "distance" between the proposed distribution and the data values. The measure

of "distance" is the value of goodness of fit statistics, which quantify the difference between the empirical distribution functions (EDF) and model cumulative distribution functions (CDF). The EDF for n data values, arranged in increasing order, is defined by the *Kth* data value having a probability of *K/n* (see Figure 1). Goodness-of-fit statistics with various EDF-CDF discrepancy weighting schemes are available.

While past researchers have used minimum distance estimation to select parameters of a statistical distribution, we proposed to use this approach to select the parameters of ARMA models that produce residuals, which best follow a normal distribution. Parr and Schucany [24] found that minimum distance estimates of normal distribution mean and variance were robust to statistical outliers. Gallagher and Moore [13] have shown that minimum distance estimation on the location parameter and maximum likelihood on the shape and scale parameters of the Weibull distribution is preferred over MLE of all three parameters.

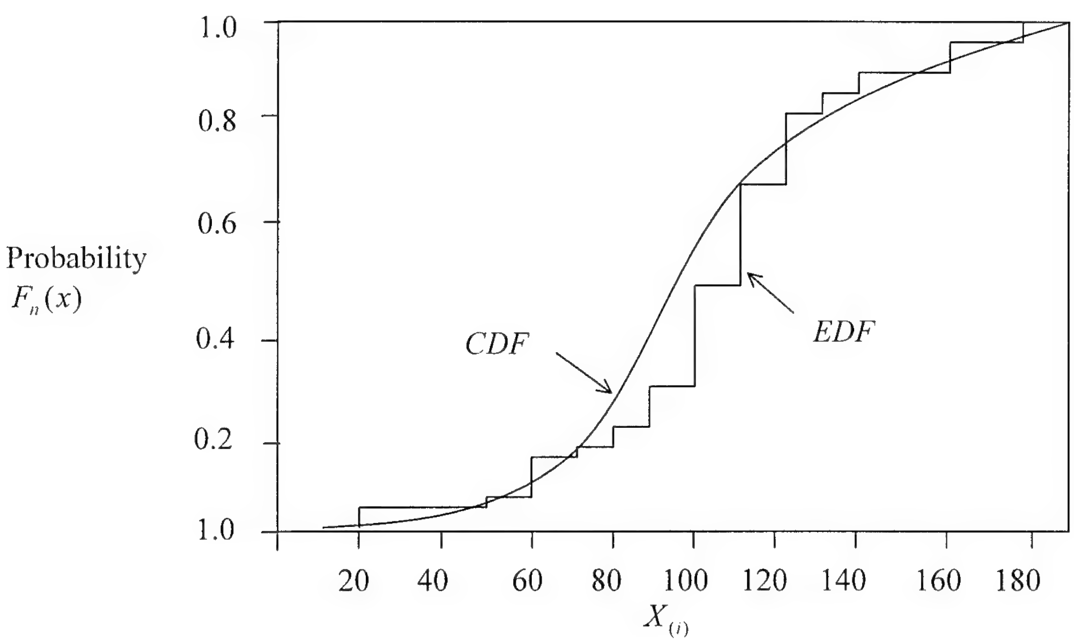


Figure 1. Empirical and Cumulative Density Functions

2.4. Goodness-of-Fit Statistics

A goodness-of-fit test uses a test statistic to measure how closely data fits hypothesized model (in our case ARMA model). The first step is to make a hypothesis (guess) of the parameters. Once the candidate model parameters are selected and applied to the time series data, a set of residuals results. The CDF and EDF values can be computed for each of the ordered residuals in the sample. The goodness-of-fit statistic is a measure of the distance between the CDF and the EDF. The statistic is compared to make a decision about the hypothesized model parameters. Then, parameters that give the lowest statistical value, hence the smallest distance, are chosen as the ARIMA model parameters.

Two important goodness-of-fit statistics are Cramer-von Misses and Anderson Darling statistics. Crown's [10] dissertation on investigating different goodness-of-fit test based on Monte Carlo studies of 10,000 runs shows that these two tests appear to be the best pair of EDF statistics, and have a much simpler computation than the Shapiro-Wilks test. Cramer-von Mises, and Anderson Darling statistics may be represented in the form

$$Q = n \int_{x=-\infty}^{x=+\infty} [F_n(x) - F_o(x)]^2 \Psi(x) dF_o(x) \tag{2.6}$$

where $F_n(x)$ is the empirical distribution function (EDF), $F_o(x)$ is the hypothesized cumulative distribution function (CDF), and n is the number of data. Furthermore, $\Psi(x)$ is a weighting function. When $\Psi(x) = 1$, we obtain the Cramer-von Misses statistic W^2. Stephens [26] presents a computational formula that calculates the distance from the EDF at mid-points between the data points and the hypothesized cumulative distribution function (CDF). With the data in ascending order, Stephens [26] formula is

$$W^2 = \sum_{i=1}^{i=n} \left[F_o(x_i) - \frac{2i-1}{2n} \right]^2 + \frac{1}{12n} \tag{2.7}$$

Watson [27] proposed an adjustment to the Cramer-von Misses to correct for the sample mean. The Anderson-Darling [2] goodness-of-fit statistic uses the weighting function

$$\Psi(x) = \frac{1}{F_o(x)(1 - F_o(x))} \tag{2.8}$$

Since the CDF ranges from zero at $x = 0$ to one at $x = \infty$, the denominator starts at zero, increases, and then decreases. Therefore, the weighting function, which is the inverse of the denominator, starts at infinity, decreases, and eventually increases back to infinity. As a result, the Anderson-Darling goodness-of-fit statistic A^2 weights the tails of the distribution much heavier than the center of the distribution. Stephens [26] gives the computational formula for the Anderson Darling goodness-of-fit statistic as

$$A^2 = \left(\frac{-1}{n}\right)\sum_{i=1}^{i=n}(2i-1)[\ln(F_o(x_i)) + \ln(1 - F_o(x_i))] - n \tag{2.9}$$

Crown [10] provides a derivation of this computational formula. Bush, Woodruff, Moore and Dunne [9] investigate the power of the Cramer-Von Mises, Anderson-Darling, Kolmogorov-Smirnov, and chi-square tests for Weibull distributions with unknown location and scale parameters and known shape parameters.

2.5. Conclusion

Many of the researches on the parameter estimation use large sample sizes. For example Dent and Min [11] study use sample size $n=100$ while Hillmer and Tiao [16] note that for $n=50$ and for the seasonal case, the conditional likelihood method performs poorly. Other researchers, including Gallagher and Moore [13], show that minimum distance estimation, which selects parameters estimates that minimize either the Cramer-von Misses, or Anderson Darling goodness of fit statistics, provide better estimates than maximum likelihood estimates or least squares in

statistical distributions with small sample sizes. The Hobbs, Moore and Miller [17] study also shows that the MDE gives better estimates than the MLE.

Although there are many applications of MDE on parameter selection of statistical distributions, our survey shows that there is no study on time series prediction using MDE. Hence this proposed research develops a new parameter estimation technique and tests it to determine if minimum distance estimation has advantages over other estimation techniques. To show this, we conducted a Monte Carlo test of minimum distance estimation in selecting ARMA parameters for small samples.

III. METHODOLOGY

3.1. Introduction

This research is based on an extensive Monte Carlo analysis of a variety of estimators for parameters of ARMA processes. However, the minimum distance estimation is the basis for this research effort. The following sections will cover the Monte Carlo simulation, random data set generation, calculation of estimation functions, and statistics.

3.2. Monte Carlo Simulation Method

Monte Carlo simulation is widely used to solve problems in statistics that are not analytically tractable. A Monte Carlo simulation uses a large number of random numbers to solve problems where the passage of time plays no substantive role [20:90-91]. More specifically, Monte Carlo simulations generate a random sample from a particular distribution and use the sample to evaluate some measure of interest. This process repeats for N total samples (or trials) and combines the measures to draw a conclusion or approximate a quantity.

Because Monte Carlo results arise from raw observational data consisting of random numbers, they have varying degrees of uncertainty. This uncertainty, however, can be made fairly negligible by collecting a large number of observations [15:4-5]. Shooman [25:259] notes that in a perfect model with perfect random numbers, the error (deviations from the true value) with Monte Carlo simulation will decrease proportionally to ${}^{1}\!/\!_{\sqrt{N}}$, where N is the number of trials. Typical simulation sizes for some recent parameter estimation studies range from 1,000 to 5,000 trials. This research uses 1000 repetitions through each computation to provide consistent results within available computer time.

The Monte Carlo procedure is illustrated by the flowchart in Figure 2. The basic steps of the simulation we follow are:

1. Generate sample data that forms an ARMA (2,2) model with known parameters.
2. Determine estimates of the parameters using defined estimation techniques on each sample.
3. Forecast future values, based on the estimated parameters.
4. Calculate selected statistics of predicted values.
5. Repeat steps one through four 1,000 times.
6. Compare the performance of each estimation technique.

We use a Visual Basic 6.0 program to execute this simulation.

3.3. Random Number Generation

First, we generate the error terms (both shock and noise) with given noise and error variance in order to generate random samples from ARMA (2,2). We use Arena 3.0 simulation program to generate normally distributed random numbers with zero mean and variance of one. These N(0,1) random numbers are then used to generate ARMA (2,2) data points by the method defined by Makridakis, Wheelwright, and McGee [22:361]. To prevent *bias* toward low values of a typical sample set, the model should be *warmed up* until it appears that the effect of the artificial conditions have disappeared [19:220]. Therefore, the first 100 generated data points are discarded to deal with initialization bias. Then, remaining data is used for parameter estimation and as future values to calculate the statistics after forecasting is done.

3.4. Forecasting Techniques

We tested a variety of forecasting techniques to evaluate the robustness of our approach. The following techniques used in this study are discussed briefly except the minimum distance

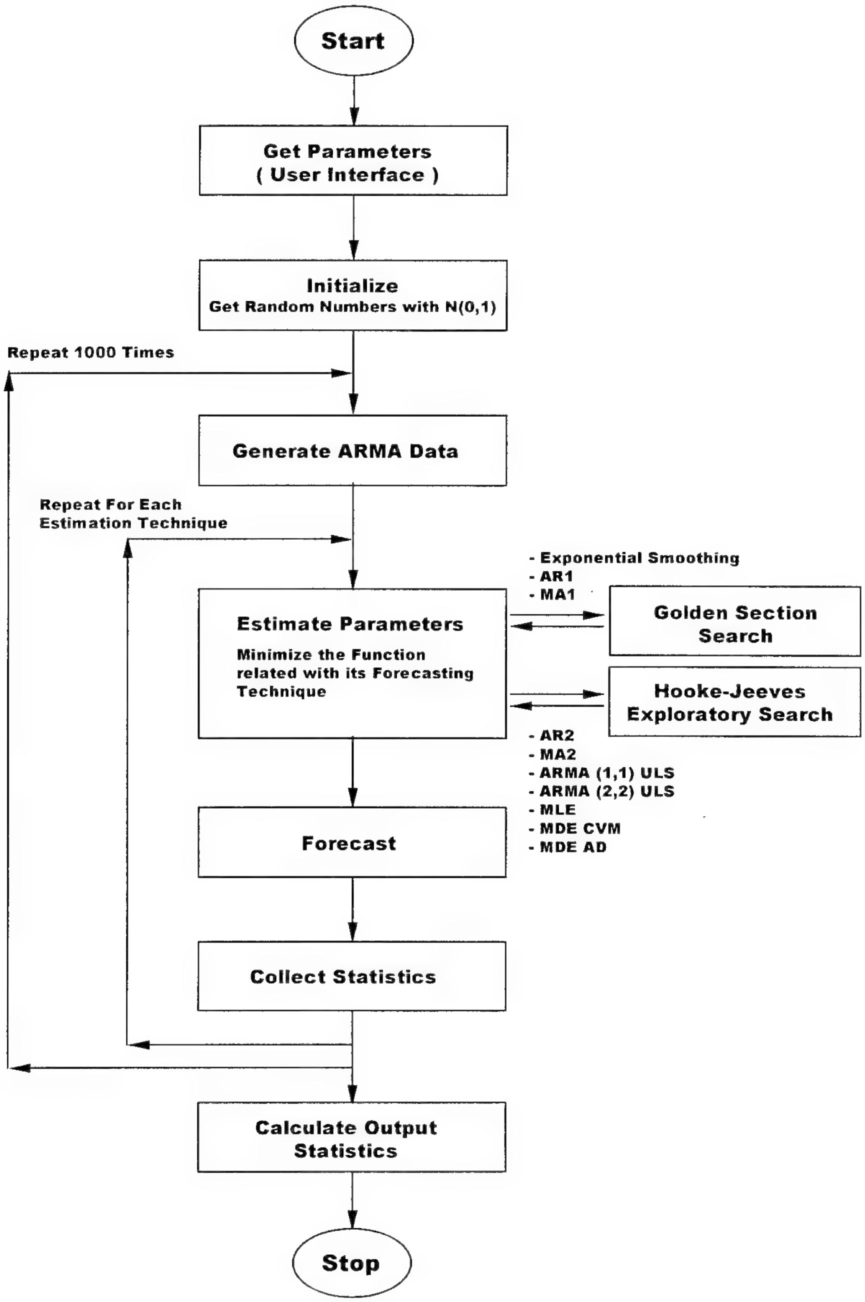


Figure 2. Flow Chart for Monte Carlo Study

methods. We leave the interested reader to look for further information on the other techniques from references such as Makridakis, Wheelwright, and McGee [22].

3.4.1. Naïve Method

Naïve method simply takes the previous observation as the current estimation; therefore the value of the *k* step ahead forecast is equal to the value of the previous observation,

$$F_{t+1}(t) = X_t \tag{3.1}$$

3.4.2. Classical Moving Average Method

In a classical moving average approach, the future observation is the current estimate of the mean, which is the average of the most *T* recent observations in a time series and is called *moving average of span T* and denoted by M_t. Thus the *k* step ahead future estimate can be represented by

$$F_{t+k}(t) = M_t = \frac{1}{T} \sum_{i=t-T+1}^{t} X_i \tag{3.2}$$

In this study we tested classical moving average forecasts with spans of size $T=5$ and $T=10$.

3.4.3. Simple Average Method

In simple average forecasting method, the process mean is estimated with the average of all the values of the time series observed to date and forecast for any future observation is simply the current estimate of the mean

$$F_{t+k}(t) = \overline{X}_t = \frac{1}{t} \sum_{i=1}^{t} X_i \tag{3.3}$$

3.4.4. Exponential Smoothing Method

In exponential smoothing case, we consider always giving the most recent observations the same amount of weight, denoted α; thus the recent values are given relatively more weight in forecasting than the older observations,

$$F_{t+k}(t) = (1-\alpha)F_t(t-1) + \alpha X_t \tag{3.4}$$

We estimate α by Golden Section Search method as described in detail in a later section.

3.4.5. Regression Method

We estimate the *slope* and *intercept* of the model, denoted by b_0 and b_1 respectively, using standard regression methods:

$$b_0 = \frac{2(2t+1)}{t(t-1)}\sum_{i=1}^{t} X_i - \frac{6}{t(t-1)}\sum_{i=1}^{t} iX_i \tag{3.5}$$

$$b_1 = \frac{12}{t(t^2-1)}\sum_{i=1}^{t} iX_i - \frac{6}{t(t-1)}\sum_{i=1}^{t} X_i \tag{3.6}$$

Since the linear trend process is postulated, the logical forecast for any future time is simply the mean predicted at that time:

$$F_{t+k}(t) = b_0 + b_1(t+k) \tag{3.7}$$

3.4.6. AR(p)

Autoregressive process is shown in Equation 2.1. In our study, we examine the autoregressive process of order *p=1* and *2,* which means the current value of the process depends on the one and two previous values respectively plus the shock term. For the first case, Equation 2.1 becomes

$$X_t = \mu' + \phi_1 X_{t-1} + \varepsilon_t \tag{3.8}$$

Here, we estimated the parameter ϕ_1 by using Golden Section Search method and μ' is calculated as

$$\mu' = (1 - \phi_1)\mu \tag{3.9}$$

and $\mu = \overline{X}$ as defined in Equation 3.3. Similarly for $p=2$, Equation 2.1 becomes

$$X_t = \mu' + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t \tag{3.10}$$

and the μ' is calculated as

$$\mu' = (1 - \phi_1 - \phi_2)\mu \tag{3.11}$$

and $\mu = \overline{X}$ as defined in Equation 3.3. The unknown parameters ϕ_1 and ϕ_2 are estimated using Hooke-Jeeves exploratory search as described in detail in a later section.

3.4.6. Box-Jenkins MA(q)

We tested the order $q=1$ and 2 for moving average models in our research. For $q=1$, the process depends on the first previous error, and Equation 2.2 reduces to

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} \tag{3.12}$$

The error terms are simply the difference between the current data and the forecasted data, and the unknown parameter θ_1 is estimated via Golden Section Search method.

For the case that $q=2$, the model for the process is

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \tag{3.13}$$

The weights θ_1 and θ_2 for the previous two errors are estimated by Hooke-Jeeves exploratory search.

3.4.7. ARMA (p, q) Unconditional Least Square (ULS)

Unconditional estimates do not assume starting values. By using Hooke-Jeeves search algorithm, we search for the model coefficients that minimize the sum of squared errors. To determine the sum of squared errors for any given set of model parameters, we must make two passes through the data. In this thesis we obtained *unconditional* estimates by *backforecasting*.

Calculation of backward forecasts is based on the concept that the autocorrelation function is symmetric due to the assumed covariance stationary [5:216-217]. Since the correlation structure is the same going forward and backward through the data, the ARMA model fits equally well in reverse time. If we start at the end of the data and backforecast, we may obtain forecasts prior to the start of the actual data. These prior observations, which decay to the mean, are used to eliminate the problem of starting values for unconditional estimation.

We previously introduced the ARMA *(p, q)* in Equation 2.3. In this case we first choose the model orders of *p=1* and *q=1*, which makes the equation

$$X_t = \mu' + \phi_1 X_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \tag{3.14}$$

the μ' is calculated by the formula given in Equation 3.9. Then we try model orders of *p=2* and *q=2*, which makes the ARMA model as

$$X_t = \mu' + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \tag{3.15}$$

here, μ' is calculated by the formula given in Equation 3.11.

For the first case, we conducted 10 backforecasts and for the second case we continued making backward forecasts until sequential backforecasts are within a tolerance or until two times the number of data points have been generated (whichever comes first).

3.4.8. ARMA (2, 2) Maximum Likelihood Estimation (MLE)

We calculate the exact likelihood function of the ARMA (2, 2) process by means of the Kalman filter. We calculate the unconditional least squares by backforecasting. By using Hooke-Jeeves' algorithm, a nonlinear search of the parameter space is conducted to find the maximum values of the functions. In addition to ARMA parameters, the measurement noise variance R should be estimated by the nonlinear search. For detailed information about likelihood function based on the Kalman filter, see Box, Jenkins, Reinsel [5], and Gardner, Harvey, Phillips [11].

3.4.9. ARMA (2, 2) Minimum Distance Estimation

We assume that error terms comes from a Normally distributed population denoted by $F(x)$ with *zero* mean and known variance σ^2 . The ARMA (2, 2) parameters that minimize the goodness-of-fit statistic are estimated through the Hooke-Jeeves algorithm. We use Stephen's formulas [26] to calculate both Cramer-von Misses W^2 and Anderson-Darling A^2 statistics as stated in Equations 2.7 and 2.9 respectively.

First, we calculate the error terms x_i , $i=1,2,...,n$, which are simply the difference between the estimated data and the actual data. Then these values are placed in ascending order ($x_1 \leq x_2 \leq ... \leq x_n$). Second, we calculate $F(x_i)$, $i=1,2,...,n$, where $F(x_i)$ is

$$F(x_i) = \int_{-3\sigma}^{x_i} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(x_i-\mu)^2\right] dx_i \tag{3.16}$$

Finally, the required W^2 and A^2 statistics are calculated from $F(x_i)$.

3.5. Nonlinear Search Methods

If the forecasting technique has one parameter to estimate, we use Golden Section Search method. Elsewhere, when the technique has more than one parameter to estimate, we prefer Hooke-Jeeves Exploratory Search method. The next two sections explain these methods.

3.5.1. Golden Section Search

We assume that the function whose parameters we are trying to estimate is *unimodal* with respect to varying location. If we are minimizing this function *f(x)*, it is said to be unimodal on [*a*, *b*] if for some point $\bar{x}$ on [*a*, *b*], *f(x)* is strictly decreasing on [*a*, $\bar{x}$] and strictly increasing on [*b*, $\bar{x}$] [28:668].

When no information is available and the objective function is unimodal, the Golden Section Search intervals reduce to zero faster than other techniques [21:136]. Let *r* be the unique positive root of the quadratic equation $r^2 + r = 1$, then $r = (5^{1/2} - 1)/2 = 0.618$.

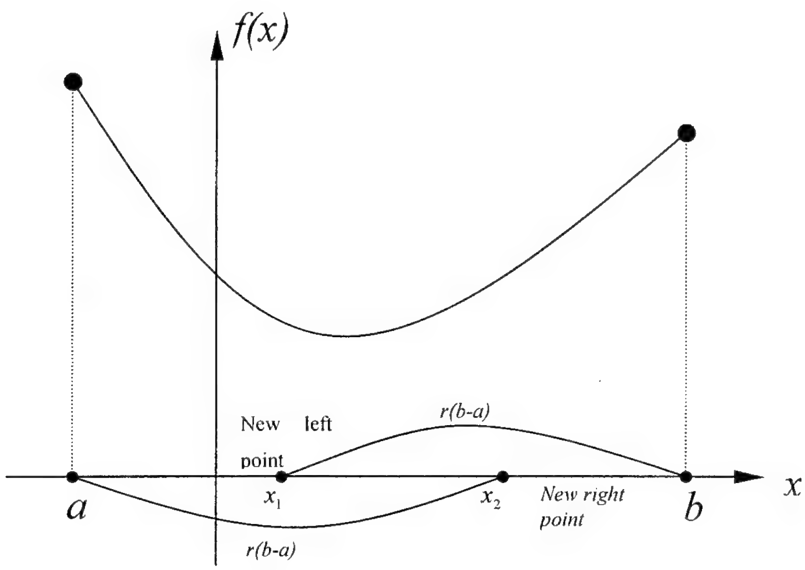


Figure 3. Golden Section Search Intervals

Golden Section Search begins by evaluating *f(x)* at points x_1 and x_2, where $x_1 = b - r(b-a)$, and $x_2 = a + r(b-a)$ (see Figure 3). Then Golden Section Search generates two new points, at which *f(x)* should again be evaluated with the following moves:

New Left-Hand Point: Move a distance equal to a fraction of r of the current interval of uncertainty from the right endpoint of the interval of uncertainty.

New Right-Hand Point: Move a distance equal to a fraction of r of the current interval of uncertainty from the left endpoint of the interval.

If $f(x_1) \geq f(x_2)$, then $\bar{x} \in (x_1, b]$, whereas if $f(x_1) < f(x_2)$, then $\bar{x} \in (a, x_2]$. If $f(x_1) \geq f(x_2)$, the reduced interval of uncertainty has length $b - x_1 = r(b-a)$, and if $f(x_1) < f(x_2)$, the reduced interval of uncertainty has a length $x_2 - a = r(b-a)$. Thus after evaluating $f(x_1)$ or $f(x_2)$, we have reduced the interval of uncertainty to a length $r(b-a)$.

Each time $f(x)$ is evaluated at two points and the interval is reduced until the difference of a and b is less than the error tolerance. An error tolerance of *0.005* is used in this research.

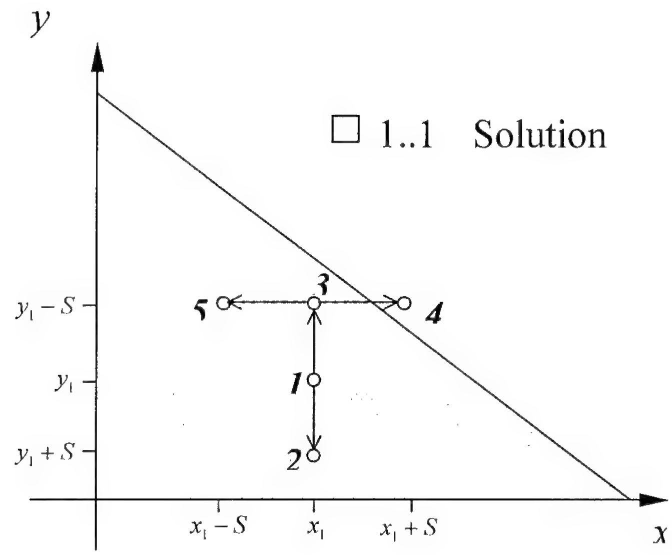
3.5.2. Hooke-Jeeves Exploratory Search

Hooke and Jeeves [18] first introduced nonlinear Hooke-Jeeves exploratory search routine. This method is a direct search algorithm that utilizes *exploratory moves* that determine an appropriate direction, and *pattern moves* that accelerate the search [7:184-185].

The method is begun by choosing an initial vector *B*, and step size h. For this research we set all the initial parameters to *zero* and *h=0.5*. Exploratory moves around *B* are made by perturbing the components of *B*, in sequence by $\pm h$ units (see Figure 4). If either perturbation improves the value of the objective function beyond the current value, the perturbed value is retained; otherwise the original value is kept. If testing each component in turn does not give a better objective, then the step size is decreased by a constant factor.

After a successful exploratory search, the algorithm makes a pattern move to a temporary vector, twice the distance from previous solution to the new solution and in that same direction, and makes exploratory moves around new point. At this point if the pattern move is successful,

another pattern search is attempted. Otherwise, the step size is reduced and another exploratory search is conducted.



Current Solution	Evaluation	Resulting Solution
1	$f(1) < f(2)$	1
1	$f(1) > f(3)$	3
3	$f(4)$ infeasible	3
3	$f(3) > f(5)$	5

Figure 4. Two Dimensional Exploratory Search

If no perturbation leads to an improvement in the objective function, a *modified pattern search* is conducted [7:185]. In this case an exhaustive search over the surface centered at B by considering all possible perturbations of the components of B by kh units, where $k= -1, 0,1$. As soon as an improvement is realized, it terminates the exhaustive search, sets the improved vector equal to B, and begins a new exploratory search on this point. The stopping rule is when the step size is below a tolerance.

3.6. Diagnostic Checking

If X_i is the actual data for time period i and F_i is the forecast (or fitted value) for the same period, then as defined in [22:44] the error can be written as

$$e_i = X_i - F_i \tag{3.17}$$

We computed the following standard measures of forecast accuracy for n forecasting terms:

$$\textit{Mean Absolute Error: } MAE = \sum_{i=1}^{n} |e_i| / n \tag{3.18}$$

$$\textit{Mean Absolute Percentage Error: } MAE = \sum_{i=1}^{n} |PE_i| / n \tag{3.19}$$

where PE_i is,

$$PE_i = \left(\frac{X_i - F_i}{X_i} \right)(100) \tag{3.20}$$

$$\textit{Sum of Squared Error: } SSE = \sum_{i=1}^{n} e_i^2 \tag{3.21}$$

We also calculate the 90 % prediction interval on one step and five steps ahead forecasts using MSE and standard prediction interval formulas, and we compute the percentage of observations fall in the one step and five steps prediction interval.

3.7 Conclusion

For the development and the implementation of minimum distance estimation technique for time series analysis, we used the methods explained above. A large set of Monte Carlo simulation were coded and run. A large portion of the coding is the previous years' effort. Numerous different parameters and error variances are tested, and the statistics are collected. The results of this effort are presented in the next chapter.

IV. RESULTS

This chapter presents the results of the research including tables summarizing the findings. For the ARMA (2,2) model with predetermined parameters, noise and error variances; random samples are generated and the results are evaluated with the criteria given in the previous chapter. For a given error and noise variance 16 different combinations of parameters are tested to cover all acceptable regions of parameters as defined by Box Jenkins [5] (See Figure 5).

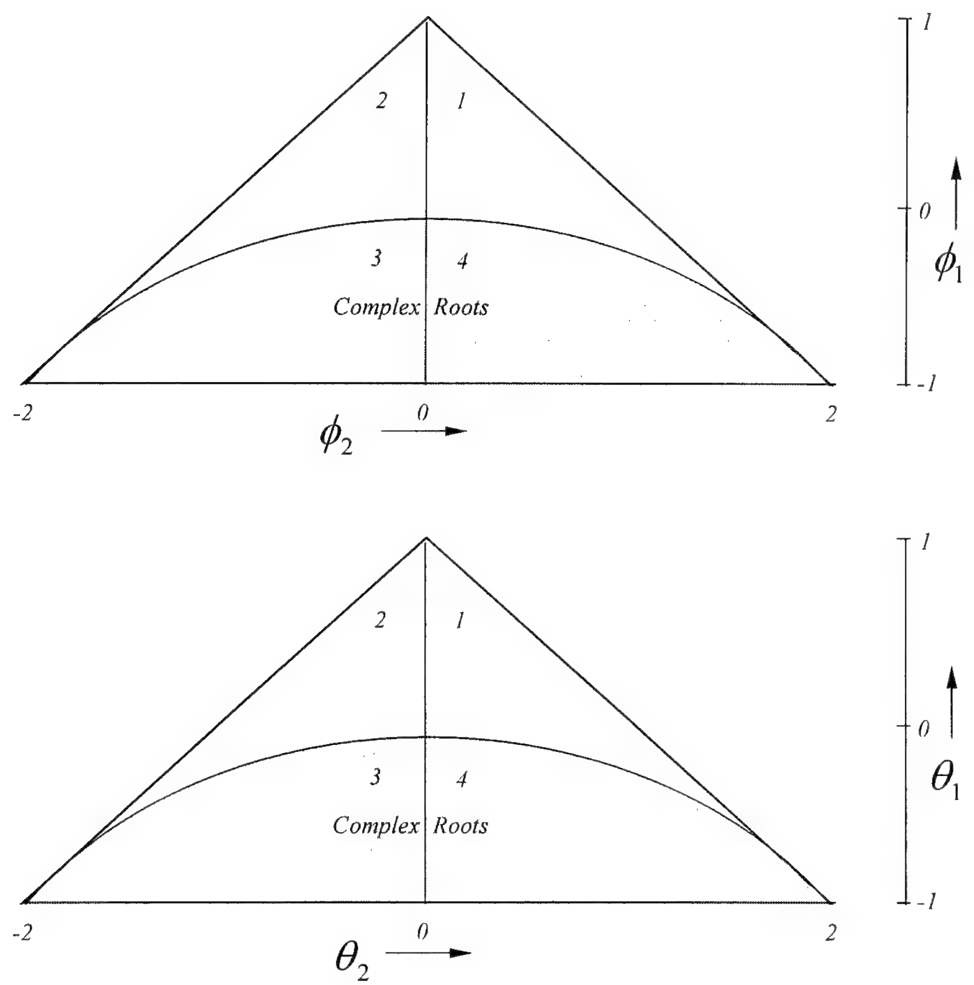


Figure 5. Acceptable Parameter Space

For each of the 16 sets of parameters, three different error and noise variances were tested. It is clearly difficult to comprehensively cover the two-dimensional stationary region with a small number of sets of parameter values. Selection of 16 cases makes no claim that these are sufficient to permit generalization to the complete space of parameters. Results for selected data samples for each case are presented in Appendix A.

It would be impossible to discuss all statistical aspects of simulation in complete detail. Therefore we concentrate on three different statistics to compare the performance of techniques. The results showing which technique performed the best are given in Table 1.

Table 1. Number of Cases an Average Is Better for ARMA (2,2) Data With Noise Across 48 Cases

	MAE	**MAPE**	**SSE**
Naïve	-	3	-
Moving Average (5)	-	-	-
Moving Average (10)	-	-	-
Simple Average	3	1	2
Exponential Smoothing	3	3	2
Regression	-	1	-
AR (1) ULS	5	5	5
AR (2) ULS	**23**	**23**	**24**
MA (1) ULS	-	1	-
MA (2) ULS	5	4	4
ARMA (1,1) ULS	1	1	1
ARMA (2,2) ULS	7	3	7
ARMA (2,2) MLE	2	3	4
ARMA (2,2) MD CVM	-	-	-
ARMA (2,2) MD AD	-	-	-

For ARMA (2,2) observed with noise data, little differentiation can be made between estimators. Comparing the estimators based on the number of times the average of MAE, MAPE,

and SSE of an estimator is the best across 48 cases; the AR(2) unconditional least squares estimator may be marginally superior to other estimators.

Within these cases, performances of the naïve, exponential smoothing, and minimum distance estimation with Cramer von Misses are the poorest. We don't advise forecasters use these three techniques to estimate an ARMA (2,2) observed with noise data. Except for these three estimators, no distinction in estimator performance is seen.

Many researchers suggest that a forecaster should choose maximum likelihood estimator if only one estimation method is available because in many cases MLE is shown to be superior to other estimators. Therefore, we compared minimum distance methods with MLE to find evidence that MDE has advantages over MLE.

For each case, we first count how many times minimum distance estimates are better than MLE out of 1000, the number of repetitions. The criteria for comparison were greater than, but not equal to, MLE. For 48 cases of ARMA (2,2) observed with noise data, the average percentage of times MDE is better than MLE is shown on Table 2.

Table 2. Percentage of Times MDE Is Better Than MLE Across 48 Cases

	MAE	**MAPE**	**SSE**
ARMA (2,2) MD CVM	37.66%	38.35%	37.64%
ARMA (2,2) MD AD	46.86%	47.35%	46.98%

These results also support our conclusion that MDE with Cramer-von Misses is not a reliable estimator. One of the things we find is that as the dynamic error and noise variances increases, MDE gives better estimates (around 50 % of the trials) than MLE (see Table 3).

Table 3. Percentage of Times MDE Is Better Than MLE Across 16 Cases For High Error Variances

	MAE	MAPE	SSE
ARMA (2,2) MD CVM	42.14%	43.67%	42.47%
ARMA (2,2) MD AD	49.33%	50.70%	49.63%

We also tested the robustness of the minimum distance approach by testing its ability to make predictions when the fitted time-series model does not match the data generation model. For this purpose we generated pure AR (2), pure MA (2) and ARMA (3,3) models and tested methods. For pure AR (2) and MA (2) processes we set noise standard deviation equal to zero and error standard deviation to a fixed value of *2.5* and ran 16 cases for each model.

Table 4. Number of Cases Out of Total 16 Cases an Average Is Better for AR (2) Model

	MAE	MAPE	SSE
Naïve	1	1	1
Moving Average (5)	-	1	-
Moving Average (10)	-	-	-
Simple Average	-	-	-
Exponential Smoothing	-	-	-
Regression	-	-	-
AR (1) ULS	1	-	1
AR (2) ULS	**11**	**14**	**12**
MA (1) ULS	3	-	2
MA (2) ULS	-	-	-
ARMA (1,1) ULS	-	-	-
ARMA (2,2) ULS	-	-	-
ARMA (2,2) MLE	-	-	-
ARMA (2,2) MD CVM	-	-	-
ARMA (2,2) MD AD	-	-	-

Table 5. Number of Cases Out of Total 16 Cases an Average Is Better for AR (2) Data

	MAE	MAPE	SSE
Naïve	-	-	-
Moving Average (5)	1	1	1
Moving Average (10)	-	-	-
Simple Average	1	-	1
Exponential Smoothing	-	1	1
Regression	-	-	-
AR (1) ULS	4	3	3
AR (2) ULS	4	4	2
MA (1) ULS	-	-	1
MA (2) ULS	-	1	-
ARMA (1,1) ULS	-	-	-
ARMA (2,2) ULS	2	1	2
ARMA (2,2) MLE	-	1	1
ARMA (2,2) MD CVM	-	-	-
ARMA (2,2) MD AD	-	-	-

Table 6. Number of Cases Out of Total 16 an Average Is Better for ARMA (3,3) Data

	MAE	MAPE	SSE
Naïve	-	-	-
Moving Average (5)	-	-	-
Moving Average (10)	-	-	-
Simple Average	-	-	-
Exponential Smoothing	-	1	-
Regression	-	-	-
AR (1) ULS	4	3	5
AR (2) ULS	-	1	-
MA (1) ULS	3	3	3
MA (2) ULS	**8**	**5**	**8**
ARMA (1,1) ULS	-	-	-
ARMA (2,2) ULS	-	2	-
ARMA (2,2) MLE	-	1	-
ARMA (2,2) MD CVM	-	-	-
ARMA (2,2) MD AD	1	-	-

We tested noise standard deviation of *0.05* and error standard deviation of *2.5* for ARMA (3,3) data and ran 12 different cases. The results of robustness tests for selected cases are presented in Appendices B, C and D respectively.

Not surprisingly, for pure AR (2) model, the AR (2) method performed best and similarly for pure MA (2) model, MA (2) method gives the best results. For ARMA (3,3) model, both autoregressive methods seem to give the best results.

The most obvious findings from these robustness tests is that the naïve, exponential smoothing and MDE with Cramer-von Misses statistic estimates should not be considered reliable. This conclusion confirmed and strengthened our findings from ARMA (2,2) tests. There is no clear picture among the remaining findings.

Since we absolutely against MDE with Cramer-von Misses statistic, we will no longer compare its results with MLE. The percentage of times MDE with Anderson Darling statistic is better than MLE for each of three cases is summarized in Table 7.

Table 7. Percentage of Times MDE AD Is Better Than MLE out of 1000

Model	**MAE**	**MAPE**	**SSE**
Pure AR (2)	47.86%	47.98%	48.10%
Pure MA (2)	49.06%	49.24%	48.69%
ARMA (3,3)	50.14%	50.26%	49.71%

These results show that MDE by using Anderson Darling statistic gives about equal results as the MLE. We also note that, in most cases, MDE AD gives better prediction interval width and coverage than MLE.

We also compared the MDE with Anderson Darling and other techniques based on the averages of MAE, MAPE and SSE. The findings are summarized on Tables 8 and 9.

Table 8. Overall Averages of 92 Cases

	MAE	*MAPE*	*SSE*	*PIW 1 MSE*	*PIC 1 MSE*	*PIW 5 MSE*	*PIC 5 MSE*
Naïve	4.84	173.20	214.08	10.46	0.89	10.45	0.89
Moving Average (T=5)	3.87	143.12	146.59	8.05	0.88	8.14	0.88
Moving Average (T=10)	3.78	142.16	139.21	7.85	0.88	8.22	0.88
Simple Average	3.72	142.91	134.40	8.00	0.89	7.95	0.89
Exponential Smoothing	4.21	151.62	172.73	8.26	0.87	8.66	0.87
Regression	3.97	140.49	154.52	7.28	0.85	7.36	0.83
AR (1) ULS	3.63	140.54	128.97	6.49	0.86	7.45	0.86
AR (2) ULS	3.33	116.20	106.37	4.72	0.84	8.46	0.89
MA (1) ULS	3.65	140.95	129.87	6.58	0.86	7.31	0.86
MA (2) ULS	3.59	137.65	132.94	5.84	0.86	7.33	0.86
ARMA (1,1) ULS	3.78	142.07	136.48	7.42	0.87	7.94	0.88
ARMA (2,2) ULS	3.41	131.05	111.60	6.32	0.87	9.03	0.86
ARMA (2,2) MLE	3.62	139.33	141.39	7.43	0.88	12.16	0.87
ARMA (2,2) MD CVM	4.22	163.12	212.66	11.04	0.91	8.09	0.87
ARMA (2,2) MD AD	3.57	130.59	124.52	6.75	0.88	7.56	0.86

Table 9. Percentages of MDE AD Averages Are Better Than Other Techniques Across Overall Cases

	MAE	***MAPE***	***SSE***
Naïve	***92.39%***	***85.87%***	***89.13%***
Moving Average (T=5)	***73.91%***	***72.83%***	***65.22%***
Moving Average (T=10)	***63.04%***	***69.57%***	***54.35%***
Simple Average	***50.00%***	***56.52%***	***44.57%***
Exponential Smoothing	***88.04%***	***80.43%***	***83.70%***
Regression	***82.61%***	***80.43%***	***77.17%***
AR (1) ULS	***38.04%***	***48.91%***	***31.52%***
AR (2) ULS	***5.43%***	***10.87%***	***4.35%***
MA (1) ULS	***41.30%***	***50.00%***	***32.61%***
MA (2) ULS	***21.74%***	***33.70%***	***20.65%***
ARMA (1,1) ULS	***54.35%***	***56.52%***	***50.00%***
ARMA (2,2) ULS	***8.70%***	***18.48%***	***6.52%***
ARMA (2,2) MLE	***48.91%***	***47.83%***	***51.09%***
ARMA (2,2) MD CVM	***98.91%***	***96.74%***	***95.65%***

These results showed that, averages of MDE with Anderson Darling is better than all other techniques except AR (2) and ARMA (2,2) ULS models but when we compare percentage of cases the average of MDE with Anderson Darling is better, the approach is only better than classical techniques. This result indicates MDE makes consistent forecasts while Box Jenkins models occasionally makes very poor forecasts.

Our findings suggest that minimum distance estimation is as good as other classical estimation methods and an equal alternative to maximum likelihood estimates. For higher error and noise variances we suggest relatively easy-to-compute minimum distance estimation method for autoregressive, moving average or mixed models.

V. CONCLUSIONS AND RECOMMENDATIONS

The inferences presented in this thesis are based on an extensive Monte Carlo study for smaller sample sizes. In particular we compare minimum distance estimates with maximum likelihood estimates. The performance of both minimum distance and maximum likelihood appear to be equal. Because of the computational complexity of maximum likelihood estimation, we recommend forecasters use minimum distance estimation. In general the Anderson Darling distance statistic is more powerful than Cramer-von Misses distance statistic to use in minimum distance estimation.

Further work should be directed to the examination of higher order autoregressive, moving average and mixed models in great detail. Performance of MDE should be compared with least squares estimation, which gave better results than MLE for very small sample sizes. Other goodness-of-fit tests can be tried to develop a more powerful minimum distance method. The performance of nonlinear search methods may be examined by giving attention to detection of global versus local optima in multi-parameter models.

Appendix A: Selected ARMA (2,2) Results

The following combinations of parameters, error and noise variances are tested and the numerical results of the selected runs (highlighted cases) are reported on the next pages.

Table 10. ARMA (2,2) Parameters

Case Number	Phi 1	Phi 2	Theta 1	Theta 2	Error σ	Noise σ
1	-0.50	0.30	0.50	0.20	1.00	0.03
2	-0.20	0.20	0.20	-0.30	1.00	0.03
3	-0.20	0.20	-1.30	-0.70	1.00	0.03
4	-0.80	0.05	-0.30	0.30	1.00	0.03
5	0.70	0.20	-0.30	0.50	1.00	0.03
6	0.20	0.60	0.20	0.10	1.00	0.03
7	0.40	0.40	0.80	-0.60	1.00	0.03
8	1.10	-0.20	-1.00	-0.40	1.00	0.03
9	0.50	-0.50	0.40	-0.70	1.00	0.03
10	1.20	-0.70	0.30	0.35	1.00	0.03
11	0.40	-0.75	-0.60	-0.30	1.00	0.03
12	0.70	-0.30	-0.10	0.70	1.00	0.03
13	-0.70	-0.65	-0.60	0.10	1.00	0.03
14	-0.35	-0.75	-0.65	-0.25	1.00	0.03
15	-0.90	-0.40	1.10	-0.60	1.00	0.03
16	-0.30	-0.50	0.60	0.25	1.00	0.03
17	-0.30	0.10	-0.40	0.45	2.50	0.05
18	-0.15	0.35	0.45	0.25	2.50	0.05
19	-0.60	0.20	0.60	-0.60	2.50	0.05
20	-0.50	0.45	-0.75	-0.55	2.50	0.05
21	0.30	0.40	-0.40	0.20	2.50	0.05
22	0.40	0.15	0.90	-0.50	2.50	0.05
23	0.70	0.05	1.00	-0.86	2.50	0.05
24	0.35	0.05	-0.30	-0.20	2.50	0.05
25	0.85	-0.45	-0.20	0.60	2.50	0.05
26	1.30	-0.80	0.20	0.25	2.50	0.05
27	0.30	-0.20	0.40	-0.60	2.50	0.05
28	0.62	-0.40	-0.80	-0.75	2.50	0.05
29	-0.50	-0.40	-0.65	0.15	2.50	0.05
30	-0.50	-0.20	0.50	0.35	2.50	0.05
31	-0.60	-0.65	0.74	-0.40	2.50	0.05
32	-1.50	-0.80	-0.15	-0.60	2.50	0.05

Table 10. (Continued)

Case Number	Phi 1	Phi 2	Theta 1	Theta 2	Error σ	Noise σ
33	-0.25	0.40	-0.10	0.15	5.00	0.10
34	-0.50	0.45	0.30	0.60	5.00	0.10
35	-0.40	0.30	0.25	-0.60	5.00	0.10
36	-0.10	0.50	-0.80	-0.30	5.00	0.10
37	0.25	0.50	-0.30	0.60	5.00	0.10
38	0.60	0.35	0.20	0.15	5.00	0.10
39	0.36	0.24	0.35	-0.45	5.00	0.10
40	0.50	0.10	-0.70	-0.42	5.00	0.10
41	1.20	-0.50	-0.20	0.30	5.00	0.10
42	0.40	-0.80	0.60	0.05	5.00	0.10
43	0.60	-0.70	0.90	-0.60	5.00	0.10
44	0.55	-0.30	-0.30	-0.34	5.00	0.10
45	-0.20	-0.10	-0.20	0.75	5.00	0.10
46	-1.20	-0.70	0.15	-0.20	5.00	0.10
47	-1.70	-0.90	-0.20	-0.80	5.00	0.10
48	-0.60	-0.80	0.20	0.60	5.00	0.10

The following abbreviations are used in this appendix:

AD : Anderson-Darling

AR : Autoregressive

ARMA : Autoregressive-Moving Average

CVM : Cramer von Misses

MA : Moving Average

MAE : Mean Absolute Error

MAPE : Mean Absolute Percentage Error

MLE : Maximum Likelihood Estimation

MSE : Mean Squared Errors

PIC : Prediction Interval Width

PIW : Prediction Interval Coverage

SSE : Sum of Squared Errors

ULS : Unconditional Least Squares

Table 11. Simulation Results $\phi_1 = -0.50,\ \phi_2 = 0.30,\ \theta_1 = 0.50,\ \theta_2 = 0.20$

TRUE PARAMETERS

Phi 1	=	-0.50	Sample size	=	25
Phi 2	=	0.30	Number of Prediction	=	5
Theta 1	=	0.50	Noise Std Deviation	=	0.03
Theta 2	=	0.20	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.19	29.24	40.96	5.91	0.87	5.13	0.86
Moving Average (T=5)	1.56	21.29	19.31	3.56	0.88	3.04	0.83
Moving Average (T=10)	1.52	20.89	18.30	3.28	0.87	3.17	0.85
Simple Average	1.51	20.73	18.06	3.45	0.91	3.03	0.86
Exponential Smoothing	1.77	24.04	26.43	3.97	0.82	3.65	0.83
Regression	1.54	21.07	18.70	3.21	0.87	3.25	0.86
AR(1)ULS	1.45	19.81	16.56	2.55	0.88	2.93	0.85
AR(2)ULS	**1.26**	**16.97**	**12.93**	1.66	0.85	3.05	0.88
MA(1)ULS	1.47	20.11	17.04	2.65	0.87	3.13	0.86
MA(2)ULS	1.42	19.41	16.18	2.08	0.87	3.14	0.86
ARMA(1,1) ULS	1.36	18.43	14.87	2.27	0.87	3.95	0.90
ARMA(2,2) ULS	1.36	18.37	14.78	2.45	0.87	3.19	0.87
ARMA(2,2) MLE	1.38	18.76	15.38	2.38	0.88	3.23	0.86
ARMA(2,2) MD CVM	1.77	23.97	31.47	4.89	0.92	3.54	0.89
ARMA(2,2) MD AD	1.39	18.88	15.61	2.63	0.90	3.19	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	300	305	286
ARMA(2,2) MD AD	494	498	484

Table 12. Simulation Results $\phi_1 = -0.70,\ \phi_2 = 0.20,\ \theta_1 = -0.30,\ \theta_2 = 0.50$

TRUE PARAMETERS

Phi 1	=	0.70	Sample size	=	25
Phi 2	=	0.20	Number of Prediction	=	5
Theta 1	=	-0.30	Noise Std Deviation	=	0.03
Theta 2	=	0.50	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	1.31	1.31	13.45	1.97	0.89	3.14	0.90
Moving Average (T=5)	1.29	1.29	13.13	2.27	0.88	3.08	0.86
Moving Average (T=10)	1.34	1.34	14.15	2.52	0.89	3.18	0.85
Simple Average	1.42	1.42	15.90	2.49	0.87	2.99	0.84
Exponential Smoothing	**1.26**	**1.26**	**12.60**	1.99	0.90	2.95	0.88
Regression	1.51	1.51	18.26	2.05	0.79	2.08	0.64
AR(1)ULS	1.37	1.37	14.93	2.02	0.84	2.33	0.76
AR(2)ULS	1.29	1.29	13.44	1.64	0.83	2.63	0.82
MA(1)ULS	1.38	1.38	15.21	2.05	0.84	2.30	0.75
MA(2)ULS	1.35	1.35	14.76	1.82	0.84	2.31	0.75
ARMA(1,1) ULS	1.45	1.45	16.92	2.26	0.86	3.34	0.85
ARMA(2,2) ULS	1.36	1.36	15.03	2.01	0.84	2.33	0.75
ARMA(2,2) MLE	1.48	1.48	24.73	6.91	0.95	2.52	0.77
ARMA(2,2) MD CVM	1.64	1.64	32.21	11.82	0.98	4.45	0.80
ARMA(2,2) MD AD	1.46	1.46	17.41	2.16	0.85	2.33	0.77

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	358	358	364
ARMA(2,2) MD AD	398	398	398

Table 13. Simulation Results $\phi_1 = -0.40,\ \phi_2 = 0.40,\ \theta_1 = 0.80,\ \theta_2 = -0.60$

TRUE PARAMETERS

Phi 1	=	0.40	Sample size	=	25
Phi 2	=	0.40	Number of Prediction	=	5
Theta 1	=	0.80	Noise Std Deviation	=	0.03
Theta 2	=	-0.60	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	1.48	2.96	17.61	3.72	0.88	3.19	0.88
Moving Average (T=5)	1.17	2.34	10.75	2.39	0.88	2.68	0.89
Moving Average (T=10)	1.20	2.41	11.42	2.46	0.88	2.79	0.88
Simple Average	1.23	2.47	11.97	2.51	0.88	2.62	0.87
Exponential Smoothing	1.25	2.50	12.50	2.61	0.85	2.64	0.87
Regression	1.30	2.61	13.35	2.22	0.83	2.24	0.78
AR(1)ULS	1.22	2.45	11.73	2.12	0.84	2.34	0.83
AR(2)ULS	1.18	2.36	11.06	1.75	0.83	2.62	0.87
MA(1)ULS	1.24	2.48	12.02	2.19	0.84	2.30	0.83
MA(2)ULS	1.21	2.42	11.52	2.00	0.85	2.30	0.83
ARMA(1,1) ULS	1.26	2.52	12.65	5.48	0.91	2.64	0.87
ARMA(2,2) ULS	**1.16**	**2.33**	**10.66**	2.32	0.90	2.32	0.82
ARMA(2,2) MLE	1.26	2.53	15.67	4.05	0.94	2.76	0.86
ARMA(2,2) MD CVM	1.47	2.94	23.41	7.89	0.98	3.80	0.87
ARMA(2,2) MD AD	1.23	2.47	12.03	2.20	0.85	2.72	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	361	361	350
ARMA(2,2) MD AD	415	417	416

Table 14. Simulation Results $\phi_1 = 1.20,\ \phi_2 = -0.70,\ \theta_1 = 0.30,\ \theta_2 = 0.35$

TRUE PARAMETERS

Phi 1	=	1.20	Sample size	=	25
Phi 2	=	-0.70	Number of Prediction	=	5
Theta 1	=	0.30	Noise Std Deviation	=	0.03
Theta 2	=	0.35	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.11	10.70	37.22	2.71	0.87	4.77	0.88
Moving Average (T=5)	1.74	8.82	23.72	3.76	0.88	2.96	0.88
Moving Average (T=10)	1.53	7.78	18.33	3.15	0.89	3.04	0.89
Simple Average	1.46	7.40	16.58	3.23	0.90	3.00	0.89
Exponential Smoothing	2.03	10.30	33.95	3.08	0.91	4.07	0.85
Regression	1.62	8.22	20.54	3.03	0.85	3.06	0.90
AR(1)ULS	1.43	7.25	15.98	2.62	0.88	3.26	0.91
AR(2)ULS	**1.24**	**6.29**	**12.16**	1.68	0.85	4.36	0.96
MA(1)ULS	1.42	7.22	15.84	2.62	0.87	2.98	0.90
MA(2)ULS	1.38	7.02	15.08	2.15	0.87	2.99	0.90
ARMA(1,1) ULS	1.83	9.24	27.04	3.15	0.87	3.28	0.86
ARMA(2,2) ULS	1.40	7.12	15.44	2.61	0.88	2.99	0.90
ARMA(2,2) MLE	1.42	7.23	16.76	2.93	0.92	3.00	0.89
ARMA(2,2) MD CVM	1.70	8.64	35.39	4.81	0.93	3.63	0.90
ARMA(2,2) MD AD	1.37	6.95	14.94	2.43	0.89	3.00	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	329	329	331
ARMA(2,2) MD AD	511	514	526

Table 15. Simulation Results $\phi_1 = 0.40,\ \phi_2 = -0.75,\ \theta_1 = -0.60,\ \theta_2 = -0.30$

TRUE PARAMETERS

Phi 1	=	0.40	Sample size	=	25
Phi 2	=	-0.75	Number of Prediction	=	5
Theta 1	=	-0.60	Noise Std Deviation	=	0.03
Theta 2	=	-0.30	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.00	30.19	33.18	3.35	0.88	3.27	0.90
Moving Average (T=5)	1.54	24.03	18.85	3.25	0.88	3.33	0.88
Moving Average (T=10)	1.50	23.51	17.92	3.21	0.88	3.30	0.89
Simple Average	1.47	22.97	17.03	3.28	0.90	3.30	0.90
Exponential Smoothing	1.77	27.05	25.66	3.43	0.89	3.12	0.91
Regression	1.55	24.15	19.28	3.05	0.85	3.08	0.87
AR(1)ULS	1.46	22.86	16.92	2.81	0.87	2.81	0.85
AR(2)ULS	**1.25**	**18.95**	**12.54**	1.66	0.85	3.26	0.91
MA(1)ULS	1.44	22.57	16.51	2.74	0.87	3.01	0.88
MA(2)ULS	1.41	21.95	15.76	2.27	0.86	3.02	0.88
ARMA(1,1) ULS	1.55	24.04	19.21	3.31	0.88	3.30	0.90
ARMA(2,2) ULS	1.38	21.48	15.30	2.80	0.89	3.03	0.88
ARMA(2,2) MLE	1.27	19.59	13.02	2.23	0.89	3.01	0.89
ARMA(2,2) MD CVM	1.79	27.85	32.73	4.18	0.89	3.92	0.88
ARMA(2,2) MD AD	1.43	22.21	16.72	2.62	0.88	3.09	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	266	267	266
ARMA(2,2) MD AD	370	362	365

Table 16. Simulation Results $\phi_1 = 0.70$, $\phi_2 = -0.30$, $\theta_1 = -0.10$, $\theta_2 = 0.70$

TRUE PARAMETERS

Phi 1	=	0.70	Sample size	=	25
Phi 2	=	-0.30	Number of Prediction	=	5
Theta 1	=	-0.10	Noise Std Deviation	=	0.03
Theta 2	=	0.70	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	1.70	10.31	23.15	2.77	0.89	3.50	0.91
Moving Average (T=5)	1.33	8.08	13.86	2.90	0.88	2.69	0.90
Moving Average (T=10)	1.24	7.55	12.08	2.68	0.87	2.62	0.90
Simple Average	1.19	7.26	11.18	2.70	0.89	2.59	0.91
Exponential Smoothing	1.54	9.35	18.98	2.79	0.89	2.94	0.90
Regression	1.28	7.83	13.05	2.52	0.85	2.55	0.90
AR(1)ULS	1.19	7.22	11.04	2.32	0.87	2.55	0.91
AR(2)ULS	1.17	7.11	10.79	1.76	0.85	3.07	0.93
MA(1)ULS	1.17	7.15	10.84	2.28	0.87	2.48	0.91
MA(2)ULS	**1.15**	**7.01**	**10.42**	1.94	0.86	2.49	0.91
ARMA(1,1) ULS	1.41	8.59	16.06	2.90	0.86	2.74	0.90
ARMA(2,2) ULS	1.17	7.11	10.79	2.82	0.92	2.50	0.91
ARMA(2,2) MLE	1.26	7.66	13.23	3.30	0.93	2.52	0.90
ARMA(2,2) MD CVM	1.44	8.77	19.15	4.00	0.95	3.18	0.90
ARMA(2,2) MD AD	1.17	7.15	11.13	2.43	0.90	2.51	0.91

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	395	396	392
ARMA(2,2) MD AD	531	533	524

Table 17. Simulation Results $\phi_1 = -0.70,\ \phi_2 = -0.65,\ \theta_1 = -0.60,\ \theta_2 = 0.10$

TRUE PARAMETERS

Phi 1	=	-0.70	Sample size	=	25
Phi 2	=	-0.65	Number of Prediction	=	5
Theta 1	=	-0.60	Noise Std Deviation	=	0.03
Theta 2	=	0.10	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	1.67	97.09	22.52	3.77	0.88	3.89	0.89
Moving Average (T=5)	1.22	80.15	11.69	2.76	0.89	2.42	0.87
Moving Average (T=10)	1.18	76.38	10.90	2.56	0.89	2.46	0.86
Simple Average	1.17	76.09	10.63	2.65	0.91	2.39	0.89
Exponential Smoothing	1.39	87.65	15.40	2.97	0.87	2.82	0.85
Regression	1.20	77.89	11.30	2.46	0.87	2.49	0.90
AR(1)ULS	1.17	74.66	10.65	2.32	0.88	2.29	0.87
AR(2)ULS	**1.05**	**51.95**	**8.71**	1.68	0.84	2.95	0.92
MA(1)ULS	1.15	73.43	10.41	2.27	0.87	2.41	0.89
MA(2)ULS	1.14	69.04	10.28	1.94	0.83	2.42	0.89
ARMA(1,1) ULS	1.19	77.58	11.08	2.52	0.89	2.48	0.90
ARMA(2,2) ULS	1.12	65.84	9.87	2.63	0.90	2.43	0.89
ARMA(2,2) MLE	1.09	57.31	9.45	2.11	0.87	2.47	0.89
ARMA(2,2) MD CVM	1.48	87.55	22.27	3.66	0.92	3.07	0.89
ARMA(2,2) MD AD	1.16	64.24	10.63	2.49	0.89	2.45	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	304	303	301
ARMA(2,2) MD AD	425	425	411

Table 18. Simulation Results $\phi_1 = -0.90,\ \phi_2 = -0.40,\ \theta_1 = 1.10,\ \theta_2 = -0.60$

TRUE PARAMETERS

Phi 1	=	-0.90	Sample size	=	25
Phi 2	=	-0.40	Number of Prediction	=	5
Theta 1	=	1.10	Noise Std Deviation	=	0.03
Theta 2	=	-0.60	Error Std Deviation	=	1.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.80	511.63	117.12	10.06	0.89	7.02	0.87
Moving Average (T=5)	2.61	347.03	53.86	5.86	0.88	5.45	0.87
Moving Average (T=10)	2.58	347.23	52.85	5.64	0.88	5.52	0.86
Simple Average	2.57	343.38	52.23	5.90	0.91	5.43	0.88
Exponential Smoothing	3.04	425.74	75.00	6.82	0.83	5.50	0.86
Regression	2.60	351.78	53.56	5.49	0.87	5.55	0.90
AR(1)ULS	2.47	325.62	48.45	4.38	0.88	5.63	0.89
AR(2)ULS	2.30	306.90	44.25	2.39	0.84	6.82	0.93
MA(1)ULS	2.49	326.44	49.13	4.49	0.88	5.36	0.88
MA(2)ULS	2.31	**286.72**	44.20	3.00	0.91	5.38	0.88
ARMA(1,1) ULS	2.62	344.53	54.78	5.13	0.87	5.56	0.88
ARMA(2,2) ULS	2.35	295.64	47.20	4.52	0.97	5.43	0.87
ARMA(2,2) MLE	**2.21**	296.18	**41.64**	3.19	0.95	6.53	0.89
ARMA(2,2) MD CVM	2.74	353.43	65.57	7.03	0.92	5.91	0.89
ARMA(2,2) MD AD	2.35	290.32	45.98	4.24	0.94	5.51	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	246	249	253
ARMA(2,2) MD AD	427	415	432

Table 19. Simulation Results $\phi_1 = -0.60,\ \phi_2 = 0.20,\ \theta_1 = 0.60,\ \theta_2 = -0.60$

TRUE PARAMETERS

Phi 1	=	-0.60	Sample size	=	25
Phi 2	=	0.20	Number of Prediction	=	5
Theta 1	=	0.60	Noise Std Deviation	=	0.05
Theta 2	=	-0.60	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	8.68	479.48	680.05	24.27	0.87	20.85	0.83
Moving Average (T=5)	6.32	373.02	316.90	14.18	0.86	12.02	0.82
Moving Average (T=10)	6.20	374.55	301.36	13.03	0.86	12.55	0.82
Simple Average	6.16	366.60	297.81	13.80	0.88	12.02	0.85
Exponential Smoothing	7.10	413.93	435.81	15.94	0.81	14.72	0.80
Regression	6.26	380.22	308.35	12.82	0.85	12.96	0.85
AR(1)ULS	5.83	351.60	268.47	9.80	0.86	11.47	0.84
AR(2)ULS	4.39	273.81	165.22	4.46	0.85	11.84	0.87
MA(1)ULS	5.94	356.92	278.61	10.33	0.85	12.46	0.85
MA(2)ULS	5.52	334.31	250.65	6.86	0.89	12.50	0.85
ARMA(1,1)ULS	6.07	365.44	289.97	11.39	0.84	13.44	0.86
ARMA(2,2) ULS	4.33	269.67	162.17	5.27	0.89	13.47	0.87
ARMA(2,2) MLE	4.83	299.75	205.18	6.39	0.89	20.47	0.90
ARMA(2,2) MD CVM	6.40	376.84	355.52	15.15	0.89	12.92	0.85
ARMA(2,2) MD AD	4.61	290.32	181.10	6.85	0.92	12.98	0.86

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	290	286	294
ARMA(2,2) MD AD	503	522	510

Table 20. Simulation Results $\phi_1 = -1.50,\ \phi_2 = -0.80,\ \theta_1 = -0.15,\ \theta_2 = -0.60$

TRUE PARAMETERS

Phi 1	=	-1.50	Sample size	=	25
Phi 2	=	-0.80	Number of Prediction	=	5
Theta 1	=	-0.15	Noise Std Deviation	=	0.05
Theta 2	=	-0.60	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	10.50	688.45	939.99	29.20	0.87	15.51	0.86
Moving Average (T=5)	7.50	317.57	444.53	16.04	0.85	15.85	0.87
Moving Average (T=10)	7.48	311.49	441.03	15.96	0.85	16.04	0.87
Simple Average	7.46	295.78	437.99	16.85	0.89	16.17	0.89
Exponential Smoothing	8.52	464.59	605.31	19.49	0.81	14.49	0.88
Regression	7.52	318.47	446.33	15.68	0.86	15.85	0.89
AR(1)ULS	7.12	294.54	399.90	12.26	0.86	17.15	0.90
AR(2)ULS	6.38	315.02	345.59	6.22	0.85	21.70	0.92
MA(1)ULS	7.21	288.68	409.02	12.70	0.86	15.28	0.87
MA(2)ULS	6.66	283.80	365.44	8.47	0.90	15.33	0.87
ARMA(1,1)ULS	7.47	288.90	439.21	14.92	0.85	15.58	0.87
ARMA(2,2) ULS	6.05	389.22	334.43	6.47	0.91	60.58	0.86
ARMA(2,2) MLE	5.73	351.49	294.13	6.53	0.89	71.59	0.95
ARMA(2,2) MD CVM	7.83	322.74	535.92	19.04	0.88	16.15	0.89
ARMA(2,2) MD AD	6.12	330.46	320.75	9.21	0.93	17.26	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	256	354	264
ARMA(2,2) MD AD	406	440	408

Table 21. Simulation Results $\phi_1 = 0.35,\ \phi_2 = 0.05,\ \theta_1 = -0.30,\ \theta_2 = -0.20$

TRUE PARAMETERS

Phi 1	=	0.35	Sample size	=	25
Phi 2	=	0.05	Number of Prediction	=	5
Theta 1	=	-0.30	Noise Std Deviation	=	0.05
Theta 2	=	-0.20	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.09	19.66	75.48	4.70	0.89	7.51	0.89
Moving Average (T=5)	3.00	19.30	70.74	5.47	0.88	6.71	0.88
Moving Average (T=10)	2.91	18.81	66.63	5.67	0.88	6.47	0.87
Simple Average	2.73	17.72	58.63	5.46	0.88	6.23	0.89
Exponential Smoothing	3.00	19.12	70.75	4.66	0.90	6.95	0.87
Regression	3.23	20.74	81.89	4.84	0.81	4.89	0.71
AR(1)ULS	2.66	17.25	55.71	4.49	0.86	5.33	0.84
AR(2)ULS	2.63	17.01	55.06	3.90	0.84	6.12	0.87
MA(1)ULS	2.67	17.37	56.47	4.58	0.86	5.07	0.82
MA(2)ULS	2.68	17.36	56.92	4.42	0.84	5.08	0.82
ARMA(1,1)ULS	2.75	17.70	59.51	4.84	0.87	5.96	0.86
ARMA(2,2) ULS	2.69	17.36	57.10	4.37	0.83	5.11	0.82
ARMA(2,2) MLE	2.76	17.83	61.26	4.74	0.83	5.15	0.83
ARMA(2,2) MD CVM	3.15	20.43	104.40	7.64	0.89	5.68	0.83
ARMA(2,2) MD AD	2.70	17.44	57.60	4.62	0.86	5.13	0.83

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	427	424	415
ARMA(2,2) MD AD	492	491	500

Table 22. Simulation Results $\phi_1 = -0.15,\ \phi_2 = 0.35,\ \theta_1 = 0.45,\ \theta_2 = -0.25$

TRUE PARAMETERS

Phi 1	=	-0.15	Sample size	=	25
Phi 2	=	0.35	Number of Prediction	=	5
Theta 1	=	0.45	Noise Std Deviation	=	0.05
Theta 2	=	0.25	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.54	34.60	99.89	8.89	0.89	7.54	0.89
Moving Average (T=5)	2.54	25.81	50.65	5.76	0.88	5.31	0.89
Moving Average (T=10)	2.47	25.35	48.17	5.47	0.89	5.37	0.89
Simple Average	2.44	24.97	46.93	5.64	0.91	5.21	0.90
Exponential Smoothing	2.90	29.12	66.97	6.36	0.86	5.73	0.87
Regression	2.51	25.64	49.60	5.24	0.88	5.30	0.91
AR(1)ULS	2.39	24.37	44.97	4.56	0.88	5.16	0.90
AR(2)ULS	2.44	24.49	46.54	4.02	0.83	5.56	0.91
MA(1)ULS	2.40	24.47	45.38	4.63	0.88	5.14	0.90
MA(2)ULS	2.42	24.53	46.43	4.39	0.85	5.16	0.90
ARMA(1,1)ULS	2.48	25.00	48.72	4.96	0.87	5.53	0.91
ARMA(2,2) ULS	2.46	24.84	47.91	5.06	0.87	5.19	0.90
ARMA(2,2) MLE	2.54	25.38	51.53	5.52	0.87	5.35	0.90
ARMA(2,2) MD CVM	2.86	28.59	78.49	7.68	0.92	5.74	0.92
ARMA(2,2) MD AD	2.53	25.51	51.37	5.45	0.87	5.20	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	437	437	437
ARMA(2,2) MD AD	512	494	523

Table 23. Simulation Results $\phi_1 = 1.30$, $\phi_2 = -0.80$, $\theta_1 = 0.20$, $\theta_2 = 0.25$

TRUE PARAMETERS

Phi 1	=	1.30	Sample size	=	25
Phi 2	=	-0.80	Number of Prediction	=	5
Theta 1	=	0.20	Noise Std Deviation	=	0.05
Theta 2	=	0.25	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	6.44	37.46	344.97	7.34	0.87	15.04	0.87
Moving Average (T=5)	5.39	32.03	229.11	11.43	0.88	8.39	0.85
Moving Average (T=10)	4.63	27.87	167.73	9.21	0.88	9.12	0.87
Simple Average	4.40	26.55	151.17	9.60	0.89	8.93	0.89
Exponential Smoothing	6.24	36.48	319.76	8.78	0.91	12.92	0.85
Regression	4.98	29.80	194.64	8.97	0.85	9.07	0.87
AR(1)ULS	4.30	25.85	144.28	7.63	0.87	9.99	0.91
AR(2)ULS	3.49	20.48	97.69	4.31	0.85	13.43	0.97
MA(1)ULS	4.29	25.80	143.59	7.66	0.87	8.88	0.88
MA(2)ULS	4.11	24.43	133.84	5.91	0.88	8.91	0.88
ARMA(1,1)ULS	4.98	29.21	196.87	9.04	0.87	9.36	0.86
ARMA(2,2) ULS	4.12	24.49	134.45	7.42	0.87	8.89	0.88
ARMA(2,2) MLE	4.10	24.05	145.68	6.66	0.88	8.79	0.87
ARMA(2,2) MD CVM	4.77	28.51	239.82	11.20	0.89	9.37	0.89
ARMA(2,2) MD AD	4.14	24.50	136.03	6.82	0.87	8.96	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	354	352	354
ARMA(2,2) MD AD	450	457	461

Table 24. Simulation Results $\phi_1 = 0.62,\ \phi_2 = -0.40,\ \theta_1 = -0.80,\ \theta_2 = -0.75$

TRUE PARAMETERS

Phi 1	= 0.62		Sample size	=	25
Phi 2	= -0.40		Number of Prediction	=	5
Theta 1	= -0.80		Noise Std Deviation	=	0.05
Theta 2	= -0.75		Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	5.66	114.31	263.59	6.83	0.87	13.03	0.88
Moving Average (T=5)	5.20	113.11	211.75	10.04	0.88	10.88	0.88
Moving Average (T=10)	4.86	114.44	185.75	9.69	0.88	10.41	0.87
Simple Average	4.52	108.22	161.05	9.43	0.89	10.14	0.90
Exponential Smoothing	5.52	113.58	247.17	8.02	0.91	11.90	0.87
Regression	5.30	119.50	221.47	8.54	0.83	8.63	0.78
AR(1)ULS	4.39	105.97	152.26	7.44	0.86	9.22	0.88
AR(2)ULS	4.19	96.55	142.20	4.54	0.84	11.50	0.92
MA(1)ULS	4.40	106.31	153.05	7.53	0.86	8.74	0.86
MA(2)ULS	4.15	100.52	140.76	5.66	0.91	8.77	0.86
ARMA(1,1)ULS	4.69	106.06	172.10	8.57	0.88	9.50	0.87
ARMA(2,2) ULS	4.17	95.02	139.87	6.25	0.89	8.79	0.85
ARMA(2,2) MLE	4.37	86.31	159.32	6.18	0.90	8.82	0.84
ARMA(2,2) MD CVM	4.86	110.24	229.52	10.95	0.88	9.24	0.86
ARMA(2,2) MD AD	4.20	93.30	142.76	6.19	0.88	8.85	0.85

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	409	407	406
ARMA(2,2) MD AD	508	501	502

Table 25. Simulation Results $\phi_1 = -0.60,\ \phi_2 = -0.65,\ \theta_1 = 0.74,\ \theta_2 = -0.40$

TRUE PARAMETERS

Phi 1	=	-0.60	Sample size	=	25
Phi 2	=	-0.65	Number of Prediction	=	5
Theta 1	=	0.74	Noise Std Deviation	=	0.05
Theta 2	=	-0.40	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	6.01	388.62	286.04	15.07	0.90	13.41	0.89
Moving Average (T=5)	4.27	327.90	142.97	9.85	0.90	8.44	0.87
Moving Average (T=10)	4.18	321.38	136.53	9.12	0.89	8.70	0.86
Simple Average	4.14	327.31	134.70	9.52	0.92	8.42	0.88
Exponential Smoothing	4.94	340.25	191.97	10.89	0.87	9.62	0.85
Regression	4.23	319.88	139.54	8.84	0.89	8.94	0.89
AR(1)ULS	4.09	318.99	131.37	7.65	0.89	8.40	0.87
AR(2)ULS	3.68	312.07	109.21	4.50	0.84	10.86	0.93
MA(1)ULS	4.06	318.59	129.57	7.59	0.89	8.64	0.88
MA(2)ULS	3.89	307.32	121.38	5.94	0.89	8.67	0.89
ARMA(1,1)ULS	4.15	328.27	135.37	8.60	0.88	8.76	0.88
ARMA(2,2) ULS	3.84	290.56	119.73	8.65	0.92	9.02	0.88
ARMA(2,2) MLE	3.70	292.05	112.47	6.10	0.90	9.16	0.89
ARMA(2,2) MD CVM	4.47	336.80	171.64	11.34	0.91	9.37	0.89
ARMA(2,2) MD AD	4.01	303.05	128.45	7.79	0.91	8.88	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	299	323	285
ARMA(2,2) MD AD	370	390	358

Table 26. Simulation Results $\phi_1 = -0.30,\ \phi_2 = 0.10,\ \theta_1 = -0.40,\ \theta_2 = 0.45$

TRUE PARAMETERS

Phi 1	=	-0.30	Sample size	=	25
Phi 2	=	0.10	Number of Prediction	=	5
Theta 1	=	-0.40	Noise Std Deviation	=	0.05
Theta 2	=	0.45	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.06	50.11	74.33	6.42	0.89	6.39	0.91
Moving Average (T=5)	2.33	40.74	42.95	5.10	0.89	4.97	0.89
Moving Average (T=10)	2.25	39.77	39.91	4.90	0.90	4.91	0.89
Simple Average	2.19	38.81	38.06	4.97	0.91	4.81	0.90
Exponential Smoothing	2.59	44.17	53.76	5.36	0.86	5.15	0.90
Regression	2.31	40.67	42.55	4.61	0.86	4.66	0.89
AR(1)ULS	2.21	39.17	38.82	4.50	0.88	4.69	0.90
AR(2)ULS	2.22	38.67	39.29	4.04	0.84	4.87	0.90
MA(1)ULS	2.19	38.94	38.35	4.45	0.87	4.55	0.88
MA(2)ULS	2.19	38.58	38.32	4.15	0.84	4.57	0.88
ARMA(1,1)ULS	2.22	39.36	39.20	4.76	0.90	4.65	0.89
ARMA(2,2) ULS	2.21	38.68	38.90	5.07	0.89	4.59	0.88
ARMA(2,2) MLE	2.24	39.21	40.20	4.89	0.87	4.63	0.88
ARMA(2,2) MD CVM	2.56	43.18	64.94	6.24	0.91	5.24	0.89
ARMA(2,2) MD AD	2.31	40.17	43.05	5.17	0.87	4.60	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	434	435	444
ARMA(2,2) MD AD	466	479	458

Table 27. Simulation Results $\phi_1 = -0.25,\ \phi_2 = 0.40,\ \theta_1 = -0.10,\ \theta_2 = 0.45$

TRUE PARAMETERS

Phi 1	=	-0.25		Sample size	=	25
Phi 2	=	0.40		Number of Prediction	=	5
Theta 1	=	-0.10		Noise Std Deviation	=	0.10
Theta 2	=	0.15		Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	5.95	385.79	282.04	14.18	0.89	13.53	0.90
Moving Average (T=5)	4.61	600.04	168.21	10.02	0.89	9.96	0.89
Moving Average (T=10)	4.49	641.81	160.12	9.70	0.89	9.97	0.89
Simple Average	4.37	616.23	152.11	9.82	0.90	9.58	0.90
Exponential Smoothing	5.04	525.03	201.84	10.55	0.86	10.64	0.88
Regression	4.69	601.46	174.15	9.02	0.86	9.11	0.86
AR(1)ULS	4.34	614.55	149.60	8.46	0.87	8.95	0.88
AR(2)ULS	4.34	423.77	149.43	7.77	0.84	9.21	0.89
MA(1)ULS	4.37	616.23	151.78	8.61	0.86	8.98	0.88
MA(2)ULS	4.43	615.99	156.90	8.76	0.84	9.01	0.88
ARMA(1,1)ULS	4.36	615.26	151.47	8.96	0.88	9.21	0.89
ARMA(2,2) ULS	4.52	457.33	163.26	9.22	0.85	9.10	0.88
ARMA(2,2) MLE	5.04	617.30	213.07	11.87	0.85	10.20	0.88
ARMA(2,2) MD CVM	5.27	620.10	379.18	13.89	0.90	9.57	0.89
ARMA(2,2) MD AD	4.54	618.89	165.66	10.15	0.88	9.36	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	553	528	561
ARMA(2,2) MD AD	580	572	591

Table 28. Simulation Results ϕ_1 = -0.10, ϕ_2 = 0.50, θ_1 = -0.80, θ_2 = -0.30

TRUE PARAMETERS

Phi 1	=	-0.10	Sample size	=	25
Phi 2	=	0.50	Number of Prediction	=	5
Theta 1	=	-0.80	Noise Std Deviation	=	0.10
Theta 2	=	-0.30	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	6.49	546.22	337.44	9.87	0.88	16.54	0.89
Moving Average (T=5)	6.67	784.48	351.79	11.61	0.87	15.38	0.86
Moving Average (T=10)	6.64	672.16	350.32	12.56	0.87	15.08	0.86
Simple Average	6.31	740.55	314.78	12.05	0.87	14.29	0.87
Exponential Smoothing	6.34	587.26	321.29	9.63	0.88	15.47	0.87
Regression	7.50	727.31	446.32	10.39	0.79	10.50	0.66
AR(1)ULS	6.12	733.52	296.93	9.81	0.85	11.74	0.81
AR(2)ULS	5.81	682.43	273.85	8.20	0.83	13.64	0.85
MA(1)ULS	6.18	737.88	303.15	10.11	0.85	11.19	0.78
MA(2)ULS	6.08	730.99	296.43	9.34	0.85	11.23	0.79
ARMA(1,1)ULS	6.21	706.48	305.45	10.78	0.86	12.14	0.81
ARMA(2,2) ULS	5.92	669.66	282.16	9.19	0.84	11.31	0.79
ARMA(2,2) MLE	6.32	656.73	329.14	10.96	0.84	11.61	0.79
ARMA(2,2) MD CVM	7.26	760.81	652.70	16.08	0.86	11.72	0.80
ARMA(2,2) MD AD	6.12	739.62	301.14	10.17	0.86	11.43	0.79

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	451	438	447
ARMA(2,2) MD AD	507	518	517

Table 29. Simulation Results $\phi_1 = 0.25,\ \phi_2 = 0.50,\ \theta_1 = -0.30,\ \theta_2 = 0.60$

TRUE PARAMETERS

Phi 1	=	0.25	Sample size	=	25
Phi 2	=	0.50	Number of Prediction	=	5
Theta 1	=	-0.30	Noise Std Deviation	=	0.10
Theta 2	=	0.60	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	5.78	14.93	261.34	10.54	0.88	12.98	0.90
Moving Average (T=5)	5.07	13.18	201.30	9.87	0.90	11.45	0.89
Moving Average (T=10)	5.03	13.12	198.19	10.13	0.89	11.44	0.87
Simple Average	4.96	12.92	192.05	10.02	0.89	10.91	0.88
Exponential Smoothing	5.27	13.66	217.30	9.54	0.89	11.44	0.88
Regression	5.50	14.29	237.47	8.89	0.84	8.99	0.77
AR(1)ULS	4.89	12.76	187.31	8.70	0.86	9.50	0.84
AR(2)ULS	4.95	12.89	192.36	8.05	0.83	9.97	0.85
MA(1)ULS	4.90	12.79	187.93	8.72	0.86	9.24	0.83
MA(2)ULS	4.95	12.91	192.56	8.55	0.83	9.28	0.83
ARMA(1,1)ULS	5.00	13.05	197.10	9.76	0.88	10.41	0.86
ARMA(2,2) ULS	4.97	12.94	194.56	9.32	0.86	9.34	0.83
ARMA(2,2) MLE	5.75	14.92	286.42	14.26	0.88	10.86	0.84
ARMA(2,2) MD CVM	5.75	14.97	411.26	13.44	0.89	9.83	0.84
ARMA(2,2) MD AD	5.11	13.29	205.57	9.60	0.85	9.38	0.83

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	528	523	540
ARMA(2,2) MD AD	559	560	558

Table 30. Simulation Results $\phi_1 = 0.36,\ \phi_2 = 0.24,\ \theta_1 = 0.35,\ \theta_2 = -0.45$

TRUE PARAMETERS

Phi 1	=	0.36	Sample size	=	25
Phi 2	=	0.24	Number of Prediction	=	5
Theta 1	=	0.35	Noise Std Deviation	=	0.10
Theta 2	=	-0.45	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	6.08	27.80	296.35	13.57	0.89	14.14	0.89
Moving Average (T=5)	5.42	25.57	233.79	10.33	0.89	12.63	0.89
Moving Average (T=10)	5.52	26.31	239.87	10.94	0.89	12.67	0.89
Simple Average	5.35	25.75	226.65	10.78	0.88	11.94	0.89
Exponential Smoothing	5.44	25.33	236.78	10.34	0.87	12.38	0.88
Regression	6.08	28.55	291.81	9.48	0.83	9.58	0.72
AR(1)ULS	5.30	25.57	223.26	9.49	0.85	10.13	0.83
AR(2)ULS	5.07	24.07	204.72	8.11	0.85	11.47	0.87
MA(1)ULS	5.36	25.82	227.86	9.75	0.84	9.94	0.83
MA(2)ULS	5.31	25.48	223.89	9.38	0.85	9.97	0.83
ARMA(1,1)ULS	5.31	25.41	224.14	9.95	0.86	10.48	0.85
ARMA(2,2) ULS	5.19	24.73	213.26	10.33	0.87	10.04	0.83
ARMA(2,2) MLE	5.64	26.64	258.43	12.93	0.87	10.63	0.84
ARMA(2,2) MD CVM	6.27	29.76	512.75	15.28	0.88	10.55	0.83
ARMA(2,2) MD AD	5.20	24.78	214.75	10.23	0.89	10.09	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	475	472	482
ARMA(2,2) MD AD	594	584	598

Table 31. Simulation Results $\phi_1 = 0.50,\ \phi_1 = 0.10,\ \theta_1 = -0.70,\ \theta_2 = -0.42$

TRUE PARAMETERS

Phi 1	=	0.50	Sample size	=	25
Phi 2	=	0.10	Number of Prediction	=	5
Theta 1	=	-0.70	Noise Std Deviation	=	0.10
Theta 2	=	-0.42	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	8.59	91.46	609.24	9.69	0.88	22.46	0.88
Moving Average (T=5)	9.49	106.40	714.12	15.56	0.87	22.01	0.85
Moving Average (T=10)	9.47	107.73	713.97	17.32	0.87	21.46	0.85
Simple Average	8.94	107.83	633.60	16.45	0.87	20.36	0.87
Exponential Smoothing	8.57	92.67	604.15	11.36	0.91	21.82	0.86
Regression	10.83	113.22	933.62	13.94	0.79	14.09	0.62
AR(1)ULS	8.57	104.51	587.82	12.65	0.85	16.06	0.79
AR(2)ULS	7.85	93.94	512.49	8.11	0.84	19.40	0.85
MA(1)ULS	8.67	105.47	601.14	13.08	0.85	15.34	0.77
MA(2)ULS	8.27	100.02	563.31	10.36	0.88	15.39	0.77
ARMA(1,1)ULS	8.54	97.96	590.84	13.52	0.87	17.92	0.81
ARMA(2,2) ULS	8.04	96.67	528.37	10.58	0.85	15.50	0.77
ARMA(2,2) MLE	8.60	95.81	654.03	12.19	0.86	16.25	0.77
ARMA(2,2) MD CVM	9.53	110.58	832.42	19.20	0.85	15.85	0.78
ARMA(2,2) MD AD	8.25	98.73	554.38	11.23	0.86	16.33	0.77

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	404	402	415
ARMA(2,2) MD AD	510	512	508

Table 32. Simulation Results $\phi_1 = 1.20,\ \phi_2 = -0.50,\ \theta_1 = -0.20,\ \theta_2 = 0.30$

TRUE PARAMETERS

Phi 1	=	1.20	Sample size	=	25
Phi 2	=	-0.50	Number of Prediction	=	5
Theta 1	=	-0.20	Noise Std Deviation	=	0.10
Theta 2	=	0.30	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	9.94	35.50	803.96	11.49	0.87	25.11	0.88
Moving Average (T=5)	9.89	36.53	771.50	17.97	0.87	20.99	0.87
Moving Average (T=10)	9.11	34.33	650.78	17.76	0.87	19.39	0.87
Simple Average	8.33	31.70	546.28	17.04	0.88	19.30	0.90
Exponential Smoothing	9.79	35.18	773.98	13.57	0.91	23.48	0.87
Regression	10.04	37.18	791.06	15.36	0.82	15.52	0.74
AR(1)ULS	8.05	30.62	512.58	13.29	0.86	17.20	0.85
AR(2)ULS	7.74	29.05	484.26	8.48	0.85	21.18	0.92
MA(1)ULS	8.10	30.82	518.54	13.55	0.86	15.82	0.83
MA(2)ULS	7.74	29.28	485.91	10.20	0.88	15.88	0.83
ARMA(1,1)ULS	8.71	31.80	603.95	14.73	0.87	18.81	0.87
ARMA(2,2) ULS	7.76	29.11	484.89	14.79	0.91	15.95	0.83
ARMA(2,2) MLE	8.55	31.55	598.64	14.88	0.90	16.22	0.82
ARMA(2,2) MD CVM	8.96	33.92	775.18	20.50	0.87	16.36	0.83
ARMA(2,2) MD AD	8.10	30.33	544.67	13.39	0.87	17.27	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	449	446	457
ARMA(2,2) MD AD	508	505	527

Table 33. Simulation Results $\phi_1 = 0.60,\ \phi_2 = -0.70,\ \theta_1 = 0.90,\ \theta_2 = -0.60$

TRUE PARAMETERS

Phi 1	=	0.60	Sample size	=	25
Phi 2	=	-0.70	Number of Prediction	=	5
Theta 1	=	0.90	Noise Std Deviation	=	0.10
Theta 2	=	-0.60	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	6.35	254.28	321.16	14.32	0.89	12.34	0.91
Moving Average (T=5)	4.63	211.66	170.27	10.31	0.89	10.14	0.90
Moving Average (T=10)	4.56	207.34	165.20	10.11	0.89	10.17	0.90
Simple Average	4.51	206.39	161.64	10.37	0.91	9.99	0.91
Exponential Smoothing	5.28	227.73	222.51	11.50	0.87	10.16	0.90
Regression	4.64	208.59	171.65	9.64	0.87	9.74	0.90
AR(1)ULS	4.54	207.76	163.70	9.24	0.87	9.82	0.90
AR(2)ULS	4.57	197.90	165.43	8.12	0.83	10.32	0.91
MA(1)ULS	4.50	206.22	161.30	9.10	0.87	9.48	0.89
MA(2)ULS	4.49	205.08	161.06	8.48	0.84	9.51	0.89
ARMA(1,1)ULS	4.53	206.97	162.96	9.69	0.88	9.59	0.89
ARMA(2,2) ULS	4.53	203.39	163.10	10.89	0.88	9.54	0.89
ARMA(2,2) MLE	4.72	203.50	179.48	11.03	0.88	9.82	0.88
ARMA(2,2) MD CVM	5.24	225.87	289.72	13.23	0.91	10.56	0.90
ARMA(2,2) MD AD	4.76	206.02	181.51	10.93	0.88	9.77	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	464	447	455
ARMA(2,2) MD AD	468	445	458

Table 34. Simulation Results $\phi_1 = -0.20,\ \phi_2 = -0.10,\ \theta_1 = -0.20,\ \theta_2 = 0.75$

TRUE PARAMETERS

Phi 1	=	-0.20	Sample size	=	25
Phi 2	=	-0.10	Number of Prediction	=	5
Theta 1	=	-0.20	Noise Std Deviation	=	0.10
Theta 2	=	0.75	Error Std Deviation	=	5.00

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	7.69	438.44	471.56	16.26	0.88	15.87	0.90
Moving Average (T=5)	5.58	478.48	247.17	12.55	0.88	11.69	0.89
Moving Average (T=10)	5.41	471.14	231.41	11.96	0.88	11.63	0.89
Simple Average	5.33	461.12	224.41	12.24	0.92	11.47	0.90
Exponential Smoothing	6.40	436.61	328.58	13.56	0.86	12.40	0.89
Regression	5.48	455.45	239.42	11.42	0.86	11.54	0.90
AR(1)ULS	5.39	464.57	229.51	11.07	0.87	11.32	0.90
AR(2)ULS	5.33	378.61	226.47	8.87	0.85	12.65	0.90
MA(1)ULS	5.33	460.61	224.22	10.84	0.87	11.18	0.89
MA(2)ULS	5.19	450.90	213.45	9.00	0.86	11.22	0.89
ARMA(1,1)ULS	5.35	461.32	225.51	11.47	0.88	11.33	0.89
ARMA(2,2) ULS	5.10	409.33	207.68	11.67	0.91	11.27	0.89
ARMA(2,2) MLE	5.37	400.38	231.23	12.41	0.90	11.84	0.89
ARMA(2,2) MD CVM	5.99	478.23	354.63	14.39	0.90	11.91	0.90
ARMA(2,2) MD AD	5.44	466.67	239.69	11.96	0.88	11.38	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	448	428	444
ARMA(2,2) MD AD	469	461	475

Appendix B: Selected Pure AR (2) Results

Table 35. AR(2) Parameters

Case Number	Phi1	Phi2	Error σ
1	-0.50	0.10	2.50
2	-0.40	0.50	2.50
3	0.70	0.10	2.50
4	1.10	-0.20	2.50
5	1.00	-0.50	2.50
6	0.30	-0.70	2.50
7	-1.10	-0.50	2.50
8	-0.40	-0.20	2.50
9	-0.90	0.05	2.50
10	-0.20	0.40	2.50
11	0.35	0.45	2.50
12	0.30	0.20	2.50
13	0.30	-0.40	2.50
14	1.10	-0.70	2.50
15	-0.20	-0.50	2.50
16	-0.60	-0.30	2.50

The following abbreviations are used in this appendix:

AD : Anderson-Darling

AR : Autoregressive

ARMA : Autoregressive-Moving Average

CVM : Cramer von Misses

MA : Moving Average

MAE : Mean Absolute Error

MAPE : Mean Absolute Percentage Error

MLE : Maximum Likelihood Estimation

MSE : Mean Squared Errors

PIC : Prediction Interval Width

PIW : Prediction Interval Coverage

SSE : Sum of Squared Errors

ULS : Unconditional Least Squares

Table 36. Simulation Results $\phi_1 = -0.50,\ \phi_2 = 0.10$

TRUE PARAMETERS

Phi 1	=	-0.50	Sample size	=	25
Phi 2	=	0.10	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.48	188.31	96.93	8.85	0.88	7.48	0.88
Moving Average (T=5)	2.52	139.91	50.09	5.66	0.89	5.32	0.89
Moving Average (T=10)	2.47	134.49	48.07	5.41	0.88	5.38	0.89
Simple Average	2.43	133.81	46.72	5.57	0.90	5.20	0.90
Exponential Smoothing	2.85	161.21	65.18	6.24	0.85	5.73	0.86
Regression	2.52	137.12	50.07	5.16	0.87	5.21	0.89
AR(1)ULS	2.37	123.82	44.54	4.47	0.88	5.07	0.89
AR(2)ULS	2.40	99.60	45.33	3.89	0.84	5.50	0.90
MA(1)ULS	2.39	126.09	45.19	4.57	0.88	5.07	0.89
MA(2)ULS	2.42	124.72	46.14	4.40	0.85	5.09	0.89
ARMA(1,1)ULS	2.42	128.16	46.48	4.94	0.87	5.29	0.89
ARMA(2,2) ULS	2.47	101.59	48.04	4.74	0.86	5.14	0.88
ARMA(2,2) MLE	2.56	110.21	52.23	5.40	0.86	5.27	0.89
ARMA(2,2) MD CVM	2.82	143.28	77.29	7.61	0.91	5.75	0.90
ARMA(2,2) MD AD	2.45	120.32	47.67	5.19	0.89	5.13	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	494	485	483
ARMA(2,2) MD AD	539	524	532

Table 37. Simulation Results ϕ_1 = *1.00*, ϕ_2 = *-0.50*

TRUE PARAMETERS

Phi 1	=	1.00	Sample size	=	25
Phi 2	=	-0.50	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	4.11	21.80	136.63	5.30	0.88	9.72	0.88
Moving Average (T=5)	3.74	20.00	108.70	7.32	0.88	7.57	0.87
Moving Average (T=10)	3.42	18.41	91.28	6.88	0.88	7.23	0.87
Simple Average	3.18	17.16	79.40	6.75	0.89	7.12	0.91
Exponential Smoothing	3.99	21.20	127.65	5.92	0.90	8.77	0.87
Regression	3.71	19.88	107.77	6.16	0.83	6.23	0.80
AR(1)ULS	3.10	16.71	75.48	5.41	0.86	6.73	0.88
AR(2)ULS	3.00	16.07	71.30	3.91	0.84	8.42	0.93
MA(1)ULS	3.10	16.74	75.76	5.46	0.86	6.25	0.86
MA(2)ULS	3.03	16.32	73.04	4.64	0.86	6.27	0.86
ARMA(1,1)ULS	3.52	18.70	98.29	6.10	0.88	7.24	0.87
ARMA(2,2) ULS	3.09	16.62	75.44	4.98	0.85	6.29	0.87
ARMA(2,2) MLE	3.20	17.15	84.69	5.35	0.87	6.33	0.86
ARMA(2,2) MD CVM	3.46	18.69	109.37	8.31	0.88	6.76	0.87
ARMA(2,2) MD AD	3.04	16.32	73.42	5.08	0.86	6.30	0.86

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	413	410	410
ARMA(2,2) MD AD	516	513	524

Table 38. Simulation Results $\phi_1 = -0.40$, $\phi_2 = 0.50$

TRUE PARAMETERS

Phi 1	=	-0.40	Sample size	=	25
Phi 2	=	0.50	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	5.26	115.34	252.39	14.36	0.89	13.58	0.84
Moving Average (T=5)	3.96	99.52	125.32	8.86	0.88	7.38	0.83
Moving Average (T=10)	3.85	98.43	116.80	8.02	0.88	8.03	0.86
Simple Average	3.83	96.82	115.39	8.49	0.89	7.43	0.85
Exponential Smoothing	4.37	103.99	165.19	9.61	0.83	9.50	0.81
Regression	3.94	98.35	121.90	7.85	0.86	7.93	0.85
AR(1)ULS	3.66	93.02	105.77	6.33	0.87	6.73	0.82
AR(2)ULS	2.60	46.53	54.37	3.89	0.84	6.01	0.86
MA(1)ULS	3.73	94.74	109.73	6.65	0.87	7.67	0.86
MA(2)ULS	3.65	92.30	106.25	5.67	0.85	7.69	0.86
ARMA(1,1)ULS	3.41	82.97	93.61	6.23	0.86	9.19	0.89
ARMA(2,2) ULS	2.79	51.73	62.53	4.46	0.84	8.18	0.89
ARMA(2,2) MLE	3.10	62.63	78.04	5.57	0.85	10.27	0.88
ARMA(2,2) MD CVM	4.18	101.57	159.00	10.55	0.91	8.05	0.87
ARMA(2,2) MD AD	3.21	72.58	83.20	5.53	0.86	7.83	0.86

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	323	331	334
ARMA(2,2) MD AD	454	447	454

Table 39. Simulation Results $\phi_1 = -1.10,\ \phi_2 = -0.50$

TRUE PARAMETERS

Phi 1	=	-1.10	Sample size	=	25
Phi 2	=	-0.50	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	4.97	1605.46	198.99	13.10	0.89	9.03	0.88
Moving Average (T=5)	3.45	1053.19	94.60	7.77	0.88	7.32	0.89
Moving Average (T=10)	3.42	1012.48	92.83	7.51	0.88	7.41	0.87
Simple Average	3.39	1044.76	91.62	7.83	0.91	7.30	0.90
Exponential Smoothing	4.01	1337.65	129.37	9.00	0.84	7.26	0.88
Regression	3.44	1040.57	94.43	7.29	0.88	7.36	0.90
AR(1)ULS	3.29	1027.74	86.22	5.96	0.88	7.58	0.90
AR(2)ULS	3.16	907.39	81.24	4.03	0.85	9.31	0.94
MA(1)ULS	3.30	1031.17	86.95	6.08	0.88	7.13	0.89
MA(2)ULS	3.20	1013.75	82.65	4.77	0.88	7.15	0.89
ARMA(1,1)ULS	3.40	1036.65	92.17	6.96	0.87	7.29	0.89
ARMA(2,2) ULS	3.30	945.38	88.37	6.20	0.90	7.18	0.88
ARMA(2,2) MLE	3.23	1082.48	84.94	5.80	0.89	7.44	0.88
ARMA(2,2) MD CVM	3.69	1095.53	124.32	9.57	0.90	7.91	0.90
ARMA(2,2) MD AD	3.30	913.26	88.81	6.25	0.91	7.36	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	339	366	346
ARMA(2,2) MD AD	446	493	450

Table 40. Simulation Results $\phi_1 = -0.40,\ \phi_2 = -0.20$

TRUE PARAMETERS

Phi 1	=	-0.40	Sample size	=	25
Phi 2	=	-0.20	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.10	222.75	75.91	7.41	0.89	6.49	0.90
Moving Average (T=5)	2.26	205.85	40.62	5.10	0.89	4.84	0.90
Moving Average (T=10)	2.21	200.34	38.72	4.89	0.89	4.85	0.91
Simple Average	2.18	195.47	37.62	5.01	0.91	4.72	0.91
Exponential Smoothing	2.57	209.82	52.35	5.57	0.87	5.07	0.89
Regression	2.25	201.84	40.36	4.64	0.87	4.69	0.90
AR(1)ULS	2.16	193.98	37.12	4.28	0.88	4.67	0.90
AR(2)ULS	2.21	192.49	38.47	3.89	0.83	5.00	0.91
MA(1)ULS	2.16	194.03	36.95	4.27	0.88	4.57	0.90
MA(2)ULS	2.19	193.33	38.08	4.18	0.85	4.59	0.90
ARMA(1,1)ULS	2.19	196.32	38.14	4.65	0.89	4.65	0.90
ARMA(2,2) ULS	2.25	196.40	40.14	5.16	0.87	4.61	0.89
ARMA(2,2) MLE	2.25	195.76	40.39	4.88	0.88	4.65	0.90
ARMA(2,2) MD CVM	2.64	209.75	73.34	6.64	0.89	5.15	0.90
ARMA(2,2) MD AD	2.33	195.85	44.65	5.25	0.88	4.70	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	425	433	434
ARMA(2,2) MD AD	460	457	457

Table 41. Simulation Results $\phi_1 = -0.20,\ \phi_2 = 0.40$

TRUE PARAMETERS

Phi 1	=	-0.20	Sample size	=	25
Phi 2	=	0.40	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.17	28.43	81.48	7.83	0.89	7.27	0.89
Moving Average (T=5)	2.46	22.50	47.73	5.32	0.89	5.31	0.88
Moving Average (T=10)	2.41	22.14	45.88	5.16	0.89	5.33	0.89
Simple Average	2.34	21.61	43.72	5.24	0.90	5.10	0.90
Exponential Smoothing	2.67	24.28	57.50	5.65	0.85	5.68	0.87
Regression	2.52	22.98	49.97	4.80	0.87	4.85	0.84
AR(1)ULS	2.31	21.28	42.48	4.41	0.87	4.73	0.88
AR(2)ULS	2.27	20.75	41.07	3.88	0.84	5.02	0.89
MA(1)ULS	2.33	21.49	43.34	4.52	0.87	4.78	0.88
MA(2)ULS	2.35	21.64	44.06	4.50	0.85	4.80	0.88
ARMA(1,1)ULS	2.33	21.39	43.24	4.57	0.87	5.18	0.89
ARMA(2,2) ULS	2.34	21.45	43.74	4.72	0.86	4.84	0.88
ARMA(2,2) MLE	2.62	23.79	57.33	5.86	0.85	5.14	0.88
ARMA(2,2) MD CVM	2.93	26.64	104.09	7.60	0.90	5.53	0.88
ARMA(2,2) MD AD	2.39	21.86	45.32	5.07	0.87	4.84	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	481	477	479
ARMA(2,2) MD AD	577	579	584

Table 42. Simulation Results $\phi_1 = 0.30,\ \phi_2 = 0.20$

TRUE PARAMETERS

Phi 1	=	0.30	Sample size	=	25
Phi 2	=	0.20	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.70	13.91	57.40	5.17	0.89	6.31	0.89
Moving Average (T=5)	2.43	12.57	46.49	4.69	0.89	5.50	0.88
Moving Average (T=10)	2.39	12.40	44.96	4.82	0.88	5.39	0.89
Simple Average	2.28	11.86	41.13	4.72	0.89	5.16	0.90
Exponential Smoothing	2.48	12.79	48.44	4.48	0.89	5.55	0.88
Regression	2.61	13.51	53.68	4.21	0.83	4.26	0.76
AR(1)ULS	2.26	11.73	40.23	4.13	0.86	4.53	0.86
AR(2)ULS	2.25	11.69	40.19	3.86	0.84	4.78	0.87
MA(1)ULS	2.27	11.78	40.60	4.19	0.86	4.36	0.84
MA(2)ULS	2.31	11.97	42.08	4.25	0.83	4.37	0.84
ARMA(1,1)ULS	2.33	12.12	43.25	4.54	0.88	4.97	0.87
ARMA(2,2) ULS	2.31	11.98	42.15	4.22	0.84	4.40	0.84
ARMA(2,2) MLE	2.53	13.13	54.14	5.43	0.85	4.46	0.84
ARMA(2,2) MD CVM	2.78	14.48	85.95	6.95	0.90	5.00	0.85
ARMA(2,2) MD AD	2.31	12.00	42.38	4.50	0.87	4.42	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	459	463	461
ARMA(2,2) MD AD	550	549	555

Table 43. Simulation Results $\phi_1 = -0.20,\ \phi_2 = -0.50$

TRUE PARAMETERS

Phi 1	=	-0.20	Sample size	=	25
Phi 2	=	-0.50	Number of Prediction	=	5
			Noise Std Deviation	=	0.00
			Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.33	224.20	88.34	7.36	0.90	7.33	0.89
Moving Average (T=5)	2.44	198.44	47.29	5.47	0.89	5.22	0.89
Moving Average (T=10)	2.38	204.59	44.67	5.27	0.89	5.19	0.89
Simple Average	2.34	208.74	43.39	5.38	0.92	5.10	0.91
Exponential Smoothing	2.76	210.89	61.34	5.97	0.88	5.70	0.87
Regression	2.41	199.11	46.41	5.00	0.86	5.06	0.89
AR(1)ULS	2.37	209.15	44.13	4.83	0.88	4.89	0.89
AR(2)ULS	2.26	200.58	40.67	3.90	0.84	5.20	0.90
MA(1)ULS	2.34	208.56	43.30	4.73	0.88	4.91	0.90
MA(2)ULS	2.33	207.29	42.71	4.22	0.84	4.93	0.90
ARMA(1,1)ULS	2.36	209.11	44.00	5.08	0.90	4.98	0.90
ARMA(2,2) ULS	2.31	203.09	42.03	5.23	0.89	4.94	0.90
ARMA(2,2) MLE	2.31	201.52	42.20	4.65	0.87	4.93	0.90
ARMA(2,2) MD CVM	2.76	278.61	77.27	6.74	0.90	6.02	0.90
ARMA(2,2) MD AD	2.39	210.30	45.76	5.36	0.89	4.97	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	383	369	381
ARMA(2,2) MD AD	451	448	440

Appendix C: Selected Pure MA (2) Results

Table 44. MA(2) Parameters

Case Number	Theta 1	Theta 2	Error σ
1	-0.50	0.10	2.50
2	-0.40	0.50	2.50
3	0.70	0.10	2.50
4	1.10	-0.20	2.50
5	1.00	-0.50	2.50
6	0.30	-0.70	2.50
7	-1.10	-0.50	2.50
8	-0.40	-0.20	2.50
9	-0.90	0.05	2.50
10	-0.20	0.40	2.50
11	0.35	0.45	2.50
12	0.30	0.20	2.50
13	0.30	-0.40	2.50
14	-0.20	-0.30	2.50
15	-0.60	-0.30	2.50
16	-0.50	0.10	2.50

The following abbreviations are used in this appendix:

AD : Anderson-Darling

AR : Autoregressive

ARMA : Autoregressive-Moving Average

CVM : Cramer von Misses

MA : Moving Average

MAE : Mean Absolute Error

MAPE : Mean Absolute Percentage Error

MLE : Maximum Likelihood Estimation

MSE : Mean Squared Errors

PIC : Prediction Interval Width

PIW : Prediction Interval Coverage

SSE : Sum of Squared Errors

ULS : Unconditional Least Squares

Table 45. Simulation Results $\theta_1 = -0.40,\ \theta_2 = 0.50$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	-0.40	Noise Std Deviation	=	0.00
Theta 2 =	0.50	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.34	45.02	88.69	6.51	0.88	7.07	0.91
Moving Average (T=5)	2.57	37.27	52.29	5.60	0.88	5.47	0.89
Moving Average (T=10)	2.47	36.67	48.06	5.36	0.90	5.37	0.89
Simple Average	2.40	36.06	45.43	5.42	0.91	5.28	0.91
Exponential Smoothing	2.87	40.19	65.67	5.76	0.86	5.74	0.89
Regression	2.55	37.41	51.66	5.02	0.86	5.08	0.89
AR(1)ULS	2.42	36.12	46.23	4.89	0.87	5.10	0.89
AR(2)ULS	2.43	36.16	46.88	4.20	0.85	5.47	0.90
MA(1)ULS	2.39	35.76	45.29	4.80	0.87	4.97	0.89
MA(2)ULS	2.35	35.29	43.92	4.25	0.86	4.98	0.89
ARMA(1,1)ULS	2.45	36.92	47.45	5.26	0.89	5.12	0.89
ARMA(2,2) ULS	2.36	35.39	44.25	5.40	0.92	5.02	0.89
ARMA(2,2) MLE	2.47	37.30	49.50	5.39	0.88	5.08	0.88
ARMA(2,2) MD CVM	2.79	44.34	74.81	6.90	0.91	5.69	0.90
ARMA(2,2) MD AD	2.50	37.77	50.10	5.39	0.87	5.03	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	439	441	429
ARMA(2,2) MD AD	482	484	474

Table 46. Simulation Results $\theta_1 = 0.70,\ \theta_2 = 0.10$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	0.70	Noise Std Deviation	=	0.00
Theta 2 =	0.10	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.58	50.04	100.93	8.65	0.90	7.30	0.89
Moving Average (T=5)	2.54	38.40	50.93	5.81	0.89	5.36	0.90
Moving Average (T=10)	2.48	37.91	48.57	5.53	0.89	5.40	0.90
Simple Average	2.45	37.62	47.51	5.70	0.91	5.28	0.92
Exponential Smoothing	2.93	42.98	68.19	6.43	0.87	5.64	0.89
Regression	2.51	38.18	49.74	5.31	0.88	5.37	0.92
AR(1)ULS	2.43	37.23	46.53	4.77	0.88	5.33	0.92
AR(2)ULS	2.49	37.88	48.79	4.21	0.83	5.85	0.93
MA(1)ULS	2.42	37.19	46.30	4.76	0.88	5.20	0.91
MA(2)ULS	2.41	36.96	45.99	4.35	0.85	5.22	0.91
ARMA(1,1)ULS	2.54	38.50	51.08	5.25	0.88	5.40	0.91
ARMA(2,2) ULS	2.49	37.75	48.76	6.09	0.90	5.24	0.91
ARMA(2,2) MLE	2.52	38.27	50.22	5.74	0.88	5.33	0.91
ARMA(2,2) MD CVM	2.86	41.93	81.61	7.70	0.92	5.87	0.92
ARMA(2,2) MD AD	2.67	39.91	61.45	6.11	0.86	5.33	0.91

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	435	442	433
ARMA(2,2) MD AD	449	453	454

Table 47. Simulation Results $\theta_1 = 0.30,\ \theta_2 = -0.70$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	0.30	Noise Std Deviation	=	0.00
Theta 2 =	-0.70	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.47	49.87	96.26	8.49	0.88	7.46	0.89
Moving Average (T=5)	2.69	41.76	57.25	5.71	0.89	5.88	0.90
Moving Average (T=10)	2.65	41.56	55.05	5.61	0.88	5.84	0.90
Simple Average	2.56	40.72	51.98	5.67	0.89	5.64	0.90
Exponential Smoothing	2.93	44.13	68.19	6.09	0.85	6.05	0.87
Regression	2.77	42.68	60.45	5.20	0.87	5.25	0.84
AR(1)ULS	2.53	40.05	50.67	4.79	0.86	5.29	0.88
AR(2)ULS	2.60	40.00	53.38	4.23	0.84	5.87	0.90
MA(1)ULS	2.55	40.43	51.64	4.92	0.86	5.19	0.87
MA(2)ULS	2.55	39.88	51.43	4.69	0.88	5.21	0.87
ARMA(1,1)ULS	2.57	41.26	52.16	5.12	0.87	5.34	0.88
ARMA(2,2) ULS	2.54	38.93	50.87	5.63	0.88	5.24	0.87
ARMA(2,2) MLE	2.66	39.99	56.64	6.22	0.88	5.30	0.88
ARMA(2,2) MD CVM	3.06	50.02	100.59	7.75	0.89	5.77	0.87
ARMA(2,2) MD AD	2.53	39.24	50.97	5.43	0.90	5.29	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	441	435	441
ARMA(2,2) MD AD	572	580	563

Table 48. Simulation Results θ_1 = *-1.10*, θ_2 = *-0.50*

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	-1.10	Noise Std Deviation	=	0.00
Theta 2 =	-0.50	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.92	83.61	121.85	5.33	0.89	9.18	0.89
Moving Average (T=5)	3.64	89.70	104.32	6.89	0.87	7.96	0.88
Moving Average (T=10)	3.48	93.56	94.81	6.91	0.88	7.62	0.88
Simple Average	3.24	90.87	82.91	6.70	0.88	7.40	0.89
Exponential Smoothing	3.79	83.45	113.81	5.81	0.90	8.45	0.88
Regression	3.81	97.34	114.10	6.02	0.81	6.08	0.77
AR(1)ULS	3.16	88.92	78.80	5.40	0.86	6.52	0.85
AR(2)ULS	3.18	86.54	80.87	3.99	0.84	7.82	0.90
MA(1)ULS	3.17	89.40	79.25	5.44	0.86	6.21	0.83
MA(2)ULS	3.07	86.47	75.39	4.47	0.87	6.23	0.84
ARMA(1,1)ULS	3.31	84.31	86.88	6.13	0.87	6.82	0.86
ARMA(2,2) ULS	3.14	85.58	78.25	5.13	0.86	6.26	0.84
ARMA(2,2) MLE	3.20	86.94	81.18	4.91	0.86	6.28	0.84
ARMA(2,2) MD CVM	3.63	99.84	135.54	8.39	0.88	7.23	0.84
ARMA(2,2) MD AD	3.20	84.02	81.26	5.24	0.86	6.31	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	401	406	398
ARMA(2,2) MD AD	484	488	487

Table 49. Simulation Results $\theta_1 = -0.20,\ \theta_2 = 0.40$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	-0.20	Noise Std Deviation	=	0.00
Theta 2 =	0.40	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.07	38.05	75.22	6.16	0.89	6.53	0.91
Moving Average (T=5)	2.36	30.64	44.15	5.17	0.89	5.03	0.89
Moving Average (T=10)	2.27	29.88	40.70	4.96	0.89	4.95	0.90
Simple Average	2.21	29.03	38.60	5.01	0.91	4.86	0.90
Exponential Smoothing	2.63	33.49	55.30	5.36	0.87	5.27	0.89
Regression	2.34	30.62	43.57	4.65	0.87	4.70	0.89
AR(1)ULS	2.23	29.30	39.31	4.53	0.87	4.71	0.90
AR(2)ULS	2.24	29.03	39.64	3.97	0.84	4.98	0.90
MA(1)ULS	2.21	29.03	38.63	4.46	0.87	4.59	0.89
MA(2)ULS	2.19	28.67	38.19	4.10	0.85	4.61	0.90
ARMA(1,1)ULS	2.26	29.57	40.46	4.92	0.90	4.76	0.90
ARMA(2,2) ULS	2.20	28.84	38.78	4.88	0.89	4.63	0.89
ARMA(2,2) MLE	2.28	29.82	41.64	4.89	0.86	4.66	0.89
ARMA(2,2) MD CVM	2.54	32.55	67.67	6.35	0.91	5.68	0.89
ARMA(2,2) MD AD	2.33	30.31	44.59	5.05	0.88	4.65	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	449	447	450
ARMA(2,2) MD AD	475	481	476

Table 50. Simulation Results $\theta_1 = 0.30,\ \theta_2 = 0.20$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	0.30	Noise Std Deviation	=	0.00
Theta 2 =	0.20	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.04	37.77	73.32	6.95	0.90	6.36	0.91
Moving Average (T=5)	2.23	29.27	39.55	5.04	0.88	4.76	0.91
Moving Average (T=10)	2.16	28.70	37.22	4.82	0.90	4.75	0.90
Simple Average	2.13	28.27	36.01	4.92	0.92	4.64	0.91
Exponential Smoothing	2.54	32.58	51.16	5.45	0.87	4.99	0.90
Regression	2.21	29.14	38.77	4.58	0.87	4.63	0.92
AR(1)ULS	2.13	28.29	36.07	4.33	0.88	4.61	0.91
AR(2)ULS	2.17	28.59	37.19	3.96	0.83	4.81	0.91
MA(1)ULS	2.12	28.20	35.79	4.30	0.87	4.50	0.90
MA(2)ULS	2.13	28.27	36.30	4.12	0.85	4.52	0.90
ARMA(1,1)ULS	2.17	28.72	37.55	4.72	0.89	4.63	0.90
ARMA(2,2) ULS	2.18	28.82	37.78	4.99	0.88	4.54	0.90
ARMA(2,2) MLE	2.22	29.30	39.56	4.94	0.88	4.57	0.91
ARMA(2,2) MD CVM	2.55	32.75	63.10	6.50	0.91	5.51	0.91
ARMA(2,2) MD AD	2.29	30.08	42.07	5.26	0.88	4.55	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	443	438	452
ARMA(2,2) MD AD	487	483	472

Table 51. Simulation Results $\theta_1 = 0.30,\ \theta_2 = -0.40$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	0.30	Noise Std Deviation	=	0.00
Theta 2 =	-0.40	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.11	43.28	77.03	7.61	0.89	6.65	0.89
Moving Average (T=5)	2.37	35.73	44.57	5.14	0.89	5.17	0.90
Moving Average (T=10)	2.33	35.33	42.77	5.02	0.89	5.15	0.90
Simple Average	2.27	34.42	40.76	5.09	0.90	4.98	0.91
Exponential Smoothing	2.61	38.12	54.14	5.53	0.85	5.34	0.88
Regression	2.42	36.41	46.23	4.68	0.87	4.73	0.86
AR(1)ULS	2.24	33.94	39.83	4.30	0.87	4.75	0.90
AR(2)ULS	2.31	34.44	41.97	3.93	0.83	5.12	0.90
MA(1)ULS	2.26	34.11	40.36	4.39	0.87	4.66	0.90
MA(2)ULS	2.30	34.54	41.56	4.40	0.86	4.67	0.90
ARMA(1,1)ULS	2.31	34.83	42.16	4.64	0.88	4.84	0.90
ARMA(2,2) ULS	2.32	34.62	42.50	4.88	0.86	4.70	0.89
ARMA(2,2) MLE	2.51	36.57	52.10	5.68	0.85	4.78	0.89
ARMA(2,2) MD CVM	2.69	39.23	69.53	7.19	0.92	5.36	0.90
ARMA(2,2) MD AD	2.34	35.04	43.33	5.06	0.88	4.71	0.89

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	503	487	500
ARMA(2,2) MD AD	575	576	580

Table 52. Simulation Results $\theta_1 = -0.20,\ \theta_2 = -0.30$

TRUE PARAMETERS

		Sample size	=	25
		Number of Prediction	=	5
Theta 1 =	-0.20	Noise Std Deviation	=	0.00
Theta 2 =	-0.30	Error Std Deviation	=	2.50

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.77	32.16	60.63	5.53	0.89	6.31	0.90
Moving Average (T=5)	2.38	28.51	44.48	4.80	0.90	5.23	0.89
Moving Average (T=10)	2.29	27.80	41.38	4.76	0.88	5.10	0.89
Simple Average	2.18	26.65	37.65	4.70	0.89	4.93	0.91
Exponential Smoothing	2.49	29.41	48.98	4.65	0.88	5.39	0.88
Regression	2.46	29.44	47.67	4.27	0.85	4.31	0.82
AR(1)ULS	2.17	26.60	37.37	4.17	0.86	4.50	0.88
AR(2)ULS	2.19	26.68	38.02	3.92	0.83	4.70	0.89
MA(1)ULS	2.18	26.74	37.74	4.23	0.86	4.33	0.86
MA(2)ULS	2.22	27.01	39.21	4.34	0.84	4.34	0.87
ARMA(1,1)ULS	2.20	26.79	38.56	4.39	0.87	4.54	0.88
ARMA(2,2) ULS	2.24	27.17	39.65	4.45	0.85	4.37	0.87
ARMA(2,2) MLE	2.47	29.71	51.43	5.40	0.84	4.39	0.87
ARMA(2,2) MD CVM	2.87	35.06	114.57	7.14	0.89	4.95	0.88
ARMA(2,2) MD AD	2.25	27.17	40.63	4.81	0.88	4.40	0.87

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	485	478	499
ARMA(2,2) MD AD	571	579	581

Appendix D: Selected ARMA (3,3) Results

Table 53. ARMA(3,3) Parameters

Case Number	Phi 1	Phi 2	Phi 3	Theta 1	Theta 2	Theta 3	Error σ	Noise σ
1	0.30	0.40	0.20	0.10	0.10	0.70	2.50	0.05
2	0.90	-0.80	0.30	0.60	-0.70	0.30	2.50	0.05
3	-0.50	0.40	0.30	-0.30	-0.60	0.30	2.50	0.05
4	-0.30	-0.40	-0.20	0.30	0.40	-0.20	2.50	0.05
5	-0.40	0.30	-0.20	-1.10	-0.60	0.30	2.50	0.05
6	0.50	-0.40	-0.30	-0.60	0.10	-0.20	2.50	0.05
7	0.50	0.20	-0.60	0.30	0.40	0.50	2.50	0.05
8	0.80	-0.40	0.40	0.50	-0.20	-0.40	2.50	0.05
9	0.30	-0.20	-0.20	0.40	-0.60	0.40	2.50	0.05
10	0.40	0.15	0.30	0.70	-0.45	-0.20	2.50	0.05
11	-0.80	0.10	0.40	-0.30	0.30	0.40	2.50	0.05
12	-0.20	0.20	-0.20	-0.30	0.30	0.50	2.50	0.05

The following abbreviations are used in this appendix:

AD : Anderson-Darling

AR : Autoregressive

ARMA : Autoregressive-Moving Average

CVM : Cramer von Misses

MA : Moving Average

MAE : Mean Absolute Error

MAPE : Mean Absolute Percentage Error

MLE : Maximum Likelihood Estimation

MSE : Mean Squared Errors

PIC : Prediction Interval Width

PIW : Prediction Interval Coverage

SSE : Sum of Squared Errors

ULS : Unconditional Least Squares

Table 54. Simulation Results $\phi_1 = 0.30,\ \phi_2 = 0.40,\ \phi_3 = 0.20,\ \theta_1 = 0.10,\ \theta_2 = 0.10,\ \theta_3 = 0.70$

TRUE PARAMETERS

Phi 1	=	0.30	Sample size	=	25
Phi 2	=	0.40	Number of Prediction	=	5
Phi 3	=	0.20	Noise Std Deviation	=	0.05
Theta 1	=	0.10	Error Std Deviation	=	2.50
Theta 2	=	0.10			
Theta 3	=	0.70			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.04	3.04	73.31	6.32	0.89	6.77	0.89
Moving Average (T=5)	2.52	2.52	49.72	5.22	0.89	5.50	0.89
Moving Average (T=10)	2.42	2.42	46.13	5.14	0.89	5.38	0.89
Simple Average	2.31	2.31	42.16	5.10	0.90	5.21	0.92
Exponential Smoothing	2.69	2.69	57.24	5.19	0.87	5.70	0.87
Regression	2.57	2.58	52.08	4.66	0.86	4.71	0.83
AR(1)ULS	2.30	2.31	42.06	4.53	0.86	4.82	0.89
AR(2)ULS	2.32	2.33	42.62	4.23	0.83	5.03	0.90
MA(1)ULS	2.31	2.32	42.49	4.60	0.86	4.68	0.88
MA(2)ULS	2.37	2.37	44.46	4.72	0.83	4.70	0.88
ARMA(1,1)ULS	2.99	2.99	94.48	6.90	0.86	6.11	0.89
ARMA(2,2) ULS	2.36	2.37	44.59	4.87	0.86	4.73	0.88
ARMA(2,2) MLE	3.18	3.18	130.88	11.56	0.94	6.16	0.89
ARMA(2,2) MD CVM	2.85	2.85	99.90	13.85	0.97	6.04	0.90
ARMA(2,2) MD AD	2.36	2.37	44.31	5.01	0.86	4.77	0.88

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	525	523	517
ARMA(2,2) MD AD	596	598	602

Table 55. Simulation Results $\phi_1 = 0.90,\ \phi_2 = -0.80,\ \phi_3 = 0.30,\ \theta_1 = 0.60,\ \theta_2 = -0.70,\ \theta_3 = 0.30$

TRUE PARAMETERS

Phi 1	=	0.90	Sample size	=	25
Phi 2	=	-0.80	Number of Prediction	=	5
Phi 3	=	0.30	Noise Std Deviation	=	0.05
Theta 1	=	0.60	Error Std Deviation	=	2.50
Theta 2	=	-0.70			
Theta 3	=	0.30			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	2.87	17.99	65.44	5.33	0.90	6.23	0.91
Moving Average (T=5)	2.40	15.15	45.21	4.96	0.89	5.16	0.89
Moving Average (T=10)	2.30	14.60	41.52	4.82	0.88	5.04	0.90
Simple Average	2.19	13.91	37.96	4.78	0.89	4.88	0.90
Exponential Smoothing	2.59	16.29	53.18	4.77	0.88	5.30	0.89
Regression	2.44	15.44	47.02	4.37	0.85	4.42	0.84
AR(1)ULS	2.18	13.83	37.53	4.21	0.87	4.55	0.88
AR(2)ULS	2.21	14.03	38.64	3.96	0.83	4.78	0.90
MA(1)ULS	2.18	13.84	37.55	4.23	0.86	4.40	0.87
MA(2)ULS	2.23	14.14	39.32	4.32	0.85	4.42	0.87
ARMA(1,1)ULS	2.31	14.60	42.06	4.72	0.89	4.82	0.90
ARMA(2,2) ULS	2.26	14.35	40.63	4.40	0.84	4.44	0.86
ARMA(2,2) MLE	2.44	15.43	49.35	5.32	0.86	4.50	0.86
ARMA(2,2) MD CVM	2.67	16.87	75.79	6.77	0.89	5.03	0.87
ARMA(2,2) MD AD	2.30	14.50	42.15	4.74	0.85	4.45	0.86

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	477	480	488
ARMA(2,2) MD AD	512	526	523

Table 56. Simulation Results $\phi_1 = -0.40,\ \phi_2 = 0.30,\ \phi_3 = -0.20,\ \theta_1 = -1.10,\ \theta_2 = -0.60,\ \theta_3 = 0.30$

TRUE PARAMETERS

Phi 1	=	-0.40	Sample size	=	25
Phi 2	=	0.30	Number of Prediction	=	5
Phi 3	=	-0.20	Noise Std Deviation	=	0.05
Theta 1	=	-1.10	Error Std Deviation	=	2.50
Theta 2	=	-0.60			
Theta 3	=	0.30			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.43	101.11	93.78	5.24	0.89	8.42	0.90
Moving Average (T=5)	3.24	104.94	82.45	6.13	0.88	7.01	0.88
Moving Average (T=10)	3.07	105.70	74.57	6.11	0.88	6.74	0.87
Simple Average	2.87	104.09	65.18	5.93	0.89	6.53	0.90
Exponential Smoothing	3.31	101.72	86.81	5.18	0.90	7.59	0.87
Regression	3.38	108.77	89.81	5.31	0.83	5.37	0.75
AR(1)ULS	2.80	101.42	62.18	4.89	0.86	5.85	0.85
AR(2)ULS	2.80	97.31	62.35	4.24	0.84	6.75	0.89
MA(1)ULS	2.82	102.34	63.01	5.00	0.87	5.48	0.84
MA(2)ULS	2.80	99.01	62.17	4.70	0.86	5.50	0.84
ARMA(1,1)ULS	2.88	101.88	65.32	5.42	0.86	5.73	0.85
ARMA(2,2) ULS	2.78	93.51	61.81	4.75	0.85	5.54	0.84
ARMA(2,2) MLE	2.85	96.98	64.99	4.86	0.84	5.56	0.83
ARMA(2,2) MD CVM	3.28	114.49	113.57	7.51	0.88	6.30	0.86
ARMA(2,2) MD AD	2.87	97.00	65.16	5.16	0.88	5.64	0.84

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	404	402	407
ARMA(2,2) MD AD	470	486	471

Table 57. Simulation Results $\phi_1 = 0.80,\ \phi_2 = -0.40,\ \phi_3 = 0.40,\ \theta_1 = 0.50,\ \theta_2 = -0.20,\ \theta_3 = -0.40$

TRUE PARAMETERS

Phi 1	=	0.80	Sample size	=	25
Phi 2	=	-0.40	Number of Prediction	=	5
Phi 3	=	0.40	Noise Std Deviation	=	0.05
Theta 1	=	0.50	Error Std Deviation	=	2.50
Theta 2	=	-0.20			
Theta 3	=	-0.40			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.09	6.22	76.71	6.22	0.89	7.56	0.89
Moving Average (T=5)	2.87	5.78	64.99	5.23	0.89	7.00	0.88
Moving Average (T=10)	3.04	6.14	73.19	5.87	0.89	7.28	0.87
Simple Average	3.21	6.46	80.89	5.86	0.86	6.81	0.86
Exponential Smoothing	2.82	5.68	63.29	5.31	0.89	6.74	0.87
Regression	3.42	6.90	93.82	4.92	0.80	4.97	0.67
AR(1)ULS	3.15	6.35	78.10	5.11	0.83	5.47	0.79
AR(2)ULS	3.03	6.09	72.08	4.64	0.81	5.55	0.79
MA(1)ULS	3.17	6.38	78.84	5.12	0.83	5.38	0.79
MA(2)ULS	3.19	6.43	79.94	5.01	0.82	5.40	0.79
ARMA(1,1)ULS	3.08	6.21	76.15	5.95	0.87	6.14	0.82
ARMA(2,2) ULS	3.19	6.43	80.41	5.26	0.83	5.44	0.79
ARMA(2,2) MLE	3.60	7.23	127.28	10.10	0.93	6.34	0.80
ARMA(2,2) MD CVM	3.67	7.40	167.06	9.70	0.94	6.13	0.80
ARMA(2,2) MD AD	3.23	6.51	82.40	5.68	0.86	5.45	0.79

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	455	453	460
ARMA(2,2) MD AD	504	500	499

Table 58. Simulation Results $\phi_1 = 0.40,\ \phi_2 = 0.15,\ \phi_3 = 0.30,\ \theta_1 = 0.70,\ \theta_2 = -0.45,\ \theta_3 = -0.20$

TRUE PARAMETERS

Phi 1	=	0.40	Sample size	=	25
Phi 2	=	0.15	Number of Prediction	=	5
Phi 3	=	0.30	Noise Std Deviation	=	0.05
Theta 1	=	0.70	Error Std Deviation	=	2.50
Theta 2	=	-0.45			
Theta 3	=	-0.20			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.29	4.95	85.46	7.83	0.89	7.28	0.89
Moving Average (T=5)	2.71	4.09	57.98	5.42	0.89	6.38	0.89
Moving Average (T=10)	2.86	4.31	63.94	5.72	0.91	6.73	0.88
Simple Average	3.07	4.62	73.73	5.84	0.88	6.32	0.86
Exponential Smoothing	2.83	4.27	63.69	5.81	0.85	6.15	0.88
Regression	3.12	4.70	76.89	5.01	0.83	5.07	0.74
AR(1)ULS	3.05	4.59	72.94	5.13	0.83	5.41	0.81
AR(2)ULS	2.93	4.41	67.46	4.67	0.80	5.68	0.84
MA(1)ULS	3.06	4.61	73.83	5.21	0.84	5.34	0.80
MA(2)ULS	3.08	4.63	74.35	5.29	0.84	5.35	0.81
ARMA(1,1)ULS	3.03	4.55	73.30	12.55	0.91	6.12	0.86
ARMA(2,2) ULS	3.01	4.53	71.81	5.91	0.86	5.39	0.81
ARMA(2,2) MLE	4.06	6.12	172.06	13.25	0.95	7.68	0.84
ARMA(2,2) MD CVM	3.54	5.33	148.15	11.37	0.97	6.07	0.83
ARMA(2,2) MD AD	3.08	4.64	74.63	5.68	0.85	5.47	0.82

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	547	550	525
ARMA(2,2) MD AD	584	583	577

Table 59. Simulation Results $\phi_1 = -0.20,\ \phi_2 = 0.20,\ \phi_3 = -0.20,\ \theta_1 = -0.30,\ \theta_2 = 0.30,\ \theta_3 = 0.50$

TRUE PARAMETERS

Phi 1	=	-0.20	Sample size	=	25
Phi 2	=	0.20	Number of Prediction	=	5
Phi 3	=	-0.20	Noise Std Deviation	=	0.05
Theta 1	=	-0.30	Error Std Deviation	=	2.50
Theta 2	=	0.30			
Theta 3	=	0.50			

RESULTS

	MAE	MAPE	SSE	PIW 1 MSE	PIC 1 MSE	PIW 5 MSE	PIC 5 MSE
Naïve	3.54	71.97	99.69	7.20	0.90	7.74	0.90
Moving Average (T=5)	2.69	59.73	56.97	6.05	0.89	5.49	0.89
Moving Average (T=10)	2.54	57.89	51.01	5.62	0.89	5.47	0.90
Simple Average	2.47	56.46	48.21	5.70	0.91	5.32	0.90
Exponential Smoothing	3.06	65.34	74.05	6.11	0.89	6.02	0.88
Regression	2.61	58.40	54.04	5.31	0.87	5.37	0.89
AR(1)ULS	2.48	56.43	48.71	5.09	0.86	5.26	0.90
AR(2)ULS	2.49	55.57	48.79	4.74	0.82	5.37	0.90
MA(1)ULS	2.48	56.72	48.64	5.09	0.86	5.21	0.90
MA(2)ULS	2.53	56.56	50.79	5.13	0.83	5.23	0.90
ARMA(1,1)ULS	2.48	56.57	48.94	5.32	0.88	5.33	0.90
ARMA(2,2) ULS	2.52	56.25	50.98	5.93	0.86	5.27	0.90
ARMA(2,2) MLE	2.58	56.82	54.07	5.91	0.87	5.51	0.89
ARMA(2,2) MD CVM	3.05	66.29	105.09	7.84	0.89	6.03	0.90
ARMA(2,2) MD AD	2.68	58.91	58.92	6.37	0.88	5.38	0.90

NUMBER OF TIMES BETTER ESTIMATES THAN MLE OUT OF 1000

	MAE	MAPE	SSE
ARMA(2,2) MD CVM	431	421	440
ARMA(2,2) MD AD	460	461	445

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Vita

1st Lt. Hakan Tekin was born in Tokat, TURKEY. He graduated from Kuleli Military High School in Istanbul, in 1990. He entered Turkish Air Force Academy in Istanbul and majored with Electronics Engineering. He graduated from academy as a 2nd Lt. on 30 August 1994.

After completing the pilot training program in Izmir and finishing the combat readiness training in Konya, he was assigned as a F-4E pilot to the 112th Fighter Squadron in Eskisehir in October 1996. After two years of serving as a pilot he assigned to 1st Tactical Air Force Headquarter as operations planning officer. In July 1999 he assigned to 1st Combined Air Operation Center in Eskisehir as mission planning officer. In August 1999, 1st Lt. Tekin was selected for the postgraduate education program and he entered the School of Engineering, Air Force Institute of Technology, WPAFB, OH.

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14. ABSTRACT

Minimum distance estimate is a statistical parameter estimate technique that selects model parameters that minimize a good-of-fit statistic. Minimum distance estimation has been demonstrated better standard approaches, including maximum likelihood estimators and least squares, in estimating statistical distribution parameters with very small data sets. This research applies minimum distance estimation to the task of making time series predictions with very few historical observations. In a Monte Carlo analysis, we test a variety of distance measures and report the results based on many different criteria. Our analysis tests the robustness of the approach by testing its ability to make predictions when the fitted time-series model does not match the data generation model. Our analysis indicates benefits in applying minimum distance estimation when making time series prediction based on less than 30 observations.

15. SUBJECT TERMS

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